Unit-I PDE

① Form the P.D.E. by eliminating the arbitrary functions from the following:

a) $z = f(x^2 - y^2)$

b) $z = \phi(x) \cdot \psi(y)$

c) $z = x + y + f(xy)$

② Solve the following P.D.E.

a) $y^2 p - xyq = x(z - 2y)$ (U.P.T.U – 2014)

b) $x^2 p + y^2 q = (x+y) z$ (U.P.T.U – 2015)

c) $pz - qz = z^2 + (x+y)^2$

③ Solve the following P.D.E. $p + 3q = 5z + \tan(y - 3x)$

(A.K.T.U – 2017)

④ Solve $x^2p^2 + y^2q^2 = z^2$

⑤ Solve $p^2 - q^2 = x - y$

⑥ Solve: $(p^2 + q^2) y = qz$.

⑦ Use Cauchy's method of characteristics to solve:

$$u_x - u_y = 0 ; \quad u(x,0) = x$$

⑧ Solve

a) $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 2\frac{\partial^3 z}{\partial x \partial y^2} = 0$ b) $r = a^2 t$

⑨ Solve the linear P.D.E. $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 0$

⑩ Solve the linear P.D.E.

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin(2x + 3y)$$

(11) $r + s - 2t = \sqrt{2x+y}$ (U.P.T.U 2015)

(12) Solve the linear P.D.E.

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos 4x \cos 3y + 30(2x+y)$$

(13) Solve:

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x.$$

(14) Solve: $(D^2 + 5DD' + 6D'^2)z = \frac{1}{y-2x}$

(15) $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$ (A.K.TU - 2018)

(16) Solve $\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x \partial y^2} - 6\frac{\partial^3 z}{\partial y^3} = \sin(x+2y) + e^{3x+y}$

(U.P.TU. - 2014)

(17) $(D^3 - 4D^2D' + 5DD'^2 - 2D'^3)z = e^{y+2x} + (y+x)^{1/2}$

(A.K.TU - 2017)

(18) Find a real function V of x and y, reducing to zero when $y=0$ and satisfying

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -4\pi(x^2+y^2)$$

(19) $(D^2 - 2DD' + 15D'^2)z = 12xy$

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Unit-II Application of PDE

① Classify the following operators:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial x \partial t}+\frac{\partial^2 u}{\partial t^2}$$

② Show that the equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$ is hyperbolic

③ Classify the P.D.E.

$$\frac{\partial^2 u}{\partial t^2}+t\frac{\partial^2 u}{\partial x \partial t}+x\frac{\partial^2 u}{\partial x^2}+2\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}+6u=0$$

④ Classify the following equation:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\frac{1}{c^2}\frac{\partial u}{\partial t}$$

⑤ Solve by the method of separation of variables:

$$4\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=3u, \quad u=3e^{-x}-e^{-5x},$$

when $t=0$

⑥ Solve the following equation by the method of separation of variables.

$$\frac{\partial^2 u}{\partial x \partial t}=e^{-t}\cos x$$

⑦ A string is stretched and fastened to two points l apart. Motion is started by the displacement of any point at a distance x from one end at a time t is given by

$$y(x,t)=A\sin\frac{\pi x}{l}\cos\frac{\pi c t}{l}.$$

(A.K.T.U.)
(2018, 2017, 2013)

(8) A string is stretched between two fixed points (0,0) and (l,0) and released rest from the initial deflection given by

$$f(x) = \begin{cases} \left(\frac{2k}{l}\right)x, & 0 < x < l/2 \\ \left(\frac{2k}{l}\right)(l-x), & \frac{l}{2} < x < l \end{cases}$$

Find the deflection of the string at any time.

(9) A tightly stretched string with fixed end points $x=0$ and $x=\pi$ is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points an initial velocity

$$\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0.03\sin x - 0.04\sin 3x$$

then find the displacement $y(x,t)$ at any point of string at any point t.