[SVNIET BBK] ASSIGNMENTS [MATHS-II] [KAS203]
[UNIT – III] [SEQUENCES AND SERIES] [2019]

1. Define Bounded and unbounded sequences with examples.
2. Define convergent, divergent and oscillating sequences with examples.
3. Define Monotonic sequences.
4. Show that every convergent sequence is bounded but converse is always not true.
5. Discuss the convergence of the following sequences $\{a_n\}$ where :

$(i)\ a_n = \frac{n+1}{n}$ $\quad (ii)\ a_n = \frac{n}{n^2+1}$ $\quad (iii)\ a_n = 1 + \frac{1}{3} + \frac{1}{3^2} + \ldots\ldots\ldots + \frac{1}{3^n}$.

6. Define Limit U_n Test.
7. Define Leibnitz's test (OR) Alternating series test with examples.
8. Examine the series: $2 + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \cdots \ldots\ldots\ldots\ldots$
9. Test the series: $\sqrt{\frac{2}{3}} + \sqrt{\frac{3}{4}} + \sqrt{\frac{4}{5}} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots$
10. Test whether the series: $\frac{1}{1+2^{-1}} + \frac{2}{1+2^{-2}} + \frac{3}{1+2^{-3}} + \cdots \ldots$ is convergent or divergent.
11. Test the convergence of the series : $\sqrt{\frac{1}{2}} + \frac{2}{\sqrt{5}} + \frac{4}{\sqrt{17}} + \frac{8}{\sqrt{65}} + \cdots \ldots\ldots\ldots\ldots + \frac{2^n}{\sqrt{4^n+1}} + \cdots \ldots$
12. Prove that the series : $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots \ldots$ is convergent.
13. Test convergence of the series(i) $2 - \frac{3}{2} + \frac{4}{3} - \frac{5}{4} + \frac{6}{5} - \cdots$ (ii) $log\frac{1}{2} - log\frac{2}{3} + log\frac{3}{4} - log\frac{4}{5} + ..$
14. ii) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots \ldots$ (iii) $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots \ldots$ (iv) $1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \cdots \ldots$
15. Define Comparison test for positive term series.
16. State $p - Test$ (OR) Hyper-Harmonic test.
17. Test the series : $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots \ldots$
18. Test the series : (i) $\frac{14}{1^3} + \frac{24}{2^3} + \frac{34}{3^3} + \cdots \ldots . + \frac{10n+4}{n^3} + \cdots \ldots\ldots$ (ii) $\frac{\sqrt{1}}{1+\sqrt{1}} + \frac{\sqrt{2}}{1+\sqrt{2}} + \frac{\sqrt{3}}{1+\sqrt{3}} + \cdots$.
   (iii) $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \cdots$ (iv) $\frac{2^p}{1^q} + \frac{3^p}{2^q} + \frac{4^p}{3^q} + \cdots \ldots$(v) $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \cdots ..$ (vi) $\sqrt{n^3+1} - \sqrt{n^3}$
   (vii) $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots \ldots$ (viii) $\frac{1}{\sqrt{2}-1} + \frac{1}{\sqrt{3}-1} + \frac{1}{\sqrt{4}-1} + \cdots \ldots$ (ix) $\frac{1}{5} + \frac{\sqrt{2}}{7} + \frac{\sqrt{3}}{9} + \frac{\sqrt{4}}{11} + \cdots \ldots$
   (x) $1 + \frac{1+2}{1+2^2} + \frac{1+3}{1+3^2} + \cdots$ (xi) $\frac{\sqrt{3}}{1.2} + \frac{\sqrt{5}}{3.4} + \frac{\sqrt{7}}{5.6} + \frac{\sqrt{9}}{7.8} + \cdots$
19. Test for convergence or divergence of the series whose n[th] term (general term) are:-
   (i) $\frac{1}{na+b}$ (ii) $\frac{2n+1}{n(n+1)(n+2)}$ (iii) $\sqrt{n+1} - \sqrt{n}$ (iv) $\sqrt{n^2+1} - n$ (v) $\sqrt{n+1} - \sqrt{n-1}$
   (vi) $\sqrt{n^2+1} - \sqrt{n^2-1}$ (vii) $\sqrt{n^3+1} - \sqrt{n^3-1}$ (vii) $(n^3+1)^{\frac{1}{3}} - n$ (viii) $\frac{1}{\sqrt{n}+\sqrt{n-1}}$
   (ix) $\frac{n}{1+n\sqrt{n+1}}$ (x) $sin\frac{1}{n}$ (xi) $cos\frac{1}{n}$ (xii) $\frac{1}{n}sin\frac{1}{n}$ (xiii)$tan^{-1}\frac{1}{n}$
20. State Cauchy's Root Test (OR) Radical Test.
21. Test the series: (i) $\sum(1 - \frac{1}{n})^{n^2}$ (ii) $(\frac{2^2}{1^2} - \frac{2}{1})^{-1} + (\frac{3^3}{2^3} - \frac{3}{2})^{-2} + (\frac{4^4}{3^4} - \frac{4}{3})^{-3} + \cdots \ldots\ldots$

   (iii) $(1 + \frac{1}{n})^{n^2}$ (iv) $(1 + \frac{1}{n})^{-n^2}$ (v) $[log(1 + \frac{1}{n})]^n$ (vi) $(1 + \frac{1}{\sqrt{n}})^{-n^{\frac{3}{2}}}$ (vii) $\sum_{n=2}^{\infty} (\frac{1}{logn})^n$

22. State D'Alembert's Test (OR) Ratio Test.

23. Test the series: (i) $\frac{2}{1^2+1}+\frac{2^2}{2^2+1}+\frac{2^3}{3^2+1}+\cdots\ldots\ldots$ (ii) $1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+\cdots\ldots+\frac{x^n}{n^2+1}+\cdots\ldots$

(iii) $\frac{1}{1.2.3}+\frac{x}{4.5.6}+\frac{x^2}{7.8.9}+\cdots..$ (iv) $\frac{1}{1+2}+\frac{2}{1+2^2}+\frac{3}{1+2^3}+\cdots.$ (v) $\frac{1!}{5}+\frac{2!}{5^2}+\frac{3!}{5^3}+\cdots\ldots$

(vi) $1+\frac{2^2}{2!}+\frac{3^2}{3!}+\frac{4^2}{4!}+\cdots..$ (vii) $1+2x+3x^2+4x^3+\cdots.$ (viii) $1^2+2^2x+3^2x^2+4^2x^3+\cdots\ldots..$

(ix) $\sum\frac{3n-1}{2^n}$ (x) $\sum\frac{x^n}{n(n+1)}$ (xi) $\sum\frac{x^n}{a+\sqrt{n}}$ (xii) $\sum\frac{1}{x^n+x^{-n}}$

24. Define Raabe's and Logarithmic Test.

25. Test the convergence of the series: $1+\frac{2}{3}.\frac{1}{4}+\frac{2}{3}.\frac{4}{5}.\frac{1}{6}+\frac{2}{3}.\frac{4}{5}.\frac{6}{7}.\frac{1}{8}+\cdots\ldots\ldots\ldots\ldots\ldots\ldots$

26. Test the convergence of the series: $1+\frac{x}{2}+\frac{1}{2}.\frac{3}{4}.x^2+\frac{1}{2}.\frac{3}{4}.\frac{5}{6}x^3+\cdots\ldots\ldots\ldots.,x>0.$

27. Test the series: $x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+\frac{5^5x^5}{5!}+\cdots\ldots\ldots....$

28. Test the series: $1+\frac{2^2}{3.4}+\frac{2^2.\ 4^2}{3.4.5.6}+\frac{2^2.4^2.6^2}{3.4.5.6.7.8}+\cdots\ldots\ldots\ldots\ldots....$

29. Test the series: $\sum\frac{1.2.3\ldots\ldots n}{4.7.10\ldots\ldots(3n+1)}x^n$ (30) $xlogx+x^2log2x+x^3log3x+\cdots..+x^n\ lognx+\cdots$

[FOURIER SERIES]

Q.1 Define Periodic Function.

Q.2Obtain the Fourier series to represent f(x) = $\frac{1}{4}(\pi-x)^2$in the interval (0, 2π) and hence obtain the following relations.

(a) $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots.. = \frac{\pi^2}{6}$ (b) $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots = \frac{\pi^2}{12}$ (c)) $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots. = \frac{\pi^2}{8}$ [G.B.T.U.2008]

Q.3 Obtain the Fourier series for f(x) = e^{-x} in the interval 0<x<2π. [G.B.T.U.2001, (SUM) 2010]

Q.4 Expand f(x) = x$\sin x$, 0<x<2π as a Fourier series. [G.B.T.U.2001,(SUM) 2010]

Q.5 Find the Fourier series for the function f(x) = x+ x^2 , -π<x<π and hence [G.B.T.U.2003, (SUM) 2010]

Show that (i) $\frac{\pi^2}{6}$ = 1+$\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\ldots.$ (ii) $\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots.$

Q.6 Express f(x) =$|x|$, -π<x<π as a Fourier series, hence show that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots. = \frac{\pi^2}{8}$ [G.B.T.U.2001]

Q.7 Obtain the Fourier series for the function f(x) =x^2, -π<x<π.sketch the graph and hence show that

(a) $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots..= \sum\frac{1}{n^2} = \frac{\pi^2}{6}$ [G.B.T.U.2001]

(b) $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots = \frac{\pi^2}{12}$ [G.B.T.U.2004]

Q.8 Expand the function f(x) = x $\sin x$ as a Fourier series in the interval -$\pi \le x \le \pi$.

Deduce that $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\frac{1}{7.9}+\ldots\ldots\ldots..=\frac{\pi-2}{4}$ [G.B.T.U.2001, 2005, 2008]

Q.9 Expand in a Fourier series the function f(x) = x in the interval 0<x<2π. [G.B.T.U.2001]

Q.10 Expand f(x) = $|\cos x|$as a Fourier series in the interval -π<x<π [G.B.T.U.2004]

Q.11 Obtain a Fourier series to represent f(x) = x^2 in the interval (0, 2π) and hence [G.B.T.U.2008]

Deduce that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots = \frac{\pi^2}{12}$

(12) Obtain Fourier series for the function $f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases}$ and hence

Show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots \ldots \ldots = \frac{\pi^2}{8}$ $[G.B.T.U.\ 2002, (CO)2010, (SUM)10]$

(13) Obtain Fourier series for the function $f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases}$ and hence

Deduce that $\frac{1}{1} - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \ldots\ldots = \frac{\pi^2}{12}$ $[G.B.T.U.(CO)2012]$

(14) Find the Fourier series for the function defined by $f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 0, & x = 0 \\ 1, & 0 < x < \pi \end{cases}$

Hence prove that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \ldots \ldots \ldots$ $[G.B.T.U.\ 2005,2012]$

(15) Obtain the Fourier series expansion of $f(x) = \left(\frac{\pi - x}{2}\right)$ for $0 < x < 2$ $[G.B.T.U.\ 2005]$

(16) Find Fourier expansion for the following $f(x) = x - x^2, \quad -1 < x < 1$ $[G.B.T.U.\ 2005]$

(17) Obtain Fourier series for function $f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2 - x), & 1 \le x \le 2 \end{cases}$ $[G.B.T.U.\ 2001,07]$

(18) Expand $\pi x - x^2$ in a half range Sine series in the interval $(0, \pi)$ upto the First three terms. $[G.B.T.U.\ 2001]$

(19) Find a series of cosine of multipliers of x which will represent xsinx in the interval $(0, \pi)$

And show that:- $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots \ldots \ldots = \frac{\pi - 2}{4}$ $[G.B.T.U.\ 2002]$

(20) Expand $f(x) = x$ as a half range

(i) Sine series in $0 < x < 2$ $[G.B.T.U.\ 2001,07]$

(ii) Cosine series in $0 < x < 2$ $[G.B.T.U.\ 2004, 07]$

(21) Obtain the half-range sine series for $f(x) = x - x^2$ is the interval $0 < x < 1$ $[GBTU\ 2012]$

(22) Define even and odd function. $[G.B.T.U.\ 2009, [G.B.T.U.\ 2009]]$

(23) Obtain the Fourier series expansion of $f(x) = \left(\frac{\pi - x}{2}\right)$ for $0 < x < 2\pi$. Deduce that .

$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \ldots \ldots \ldots$ $[G.B.T.U. 2007, 09,\ M.T.U. 2011]$

(24) Find Fourier expansion for the following $f(x) = x^3$ in $(-\pi < x < \pi)$. $[G.B.T.U.\ (SUM)2009]$

(25) Obtain Fourier series for the function $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 2\pi - x, & \pi \le x \le \pi \end{cases}$ and hence

Show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots \ldots \ldots = \frac{\pi^2}{8}$ $[G.B.T.U.(SUM)2010]$

(26) Find Fourier series for following periodic function, $f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$

Also prove that: $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \ldots \ldots \ldots$ $[G.B.T.U.\ 2009,\ 2010]$

(27) Find Fourier series for periodic function, $f(x) = \begin{cases} x^2, & -\pi < x < 0 \\ -x^2, & 0 < x < \pi \end{cases}$ $[G.B.T.U.CO\ (2009]$