

[Vision Institute Of Technology Kanpur]
[B.Tech.1st Year] [AG] [ASSIGNMENT UNIT-1)]
[Engineering Mathematics- II (KAG-201)] [2019-20]

1. Transform $\begin{bmatrix} 1 & 3 & 3 \\ 2 & 4 & 10 \\ 3 & 8 & 4 \end{bmatrix}$ into unit matrix by using elementary transformations: [GBTU 07,09, 11]

2. Find the inverse of the following matrices employing elementary transformations:

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

[G.B.T.U. 2008, (SUM) 2009]

3. Employing elementary transformations, find the inverse of the matrices $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ [GBTU 07]

4. Find the inverse of the matrices by using elementary row operations: $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ [GBTU 14]

5. Find the inverse of the matrices: $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ [GBTU 12]

6. Using elementary transformations to reduce the following matrices A to triangular form and

hence find the rank of A. $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

G.B.T.U. (C.O) 2006, G.B.T.U. 2011]

7. Find Rank of matrix .(a) $A = \begin{bmatrix} 2 & 3 & -2 & 4 \\ 3 & -2 & 1 & 2 \\ 3 & 2 & 3 & 4 \\ -2 & 4 & 0 & 5 \end{bmatrix}$ [GBTU 07] , (b) $A = \begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ [GBTU 12]

8. Reduce the matrix $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & 1 & -3 & -3 \\ 1 & 0 & 1 & 2 \\ 1 & -1 & 3 & 3 \end{bmatrix}$ to column Echelon form hence find Rank. [GBTU 10]

9. Reduce A to Echelon form and then to its row canonical form where $A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$

Hence find the rank of matrix A.

[GBTU 2015]

10. Reduce the matrix to Normal form and hence find Rank $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 3 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$ [GBTU 2010]

11. Find the Rank of the matrices using elementary transformations:

(a) $\begin{bmatrix} 1 & -3 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 3 & 4 & 1 & -2 \end{bmatrix}$ [GBTU 2010]

(b) $\begin{bmatrix} -2 & -1 & 3 & -1 \\ 1 & 2 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ [GBTU 2013]

12. Reduce the matrix to Normal form and find Rank, (a) $A = \begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$ [GBTU 2011]

$$(b) A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix} \quad [\text{GBTU 2007}]$$

$$(c) A = \begin{bmatrix} 3 & 2 & -1 \\ 4 & 2 & 6 \\ 7 & 4 & 5 \end{bmatrix} \quad [\text{AKTU 2016}]$$

$$(d) A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix} \quad [\text{GBTU 2015}]$$

$$(e) A = \begin{bmatrix} 0 & 1 & 2 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix} \quad [\text{GBTU 2011}]$$

$$(f) A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \end{bmatrix} \quad [\text{GBTU 2014}]$$

15. Find all value of P for which the matrix $A = \begin{bmatrix} 3 & P & P \\ P & 3 & P \\ P & P & 3 \end{bmatrix}$ is of rank 1. [UPTU 2012]

(16) Using matrix method, Show that the equations: $3x + 3y + 2z = 1$, $x + 2y = 4$,
 $10y + 3z = -2$, $2x - 3y - z = 5$ are consistent and hence obtain the solution for x ,
 y , z .

[G. B. T. U. (A. G.) SUM 2010, U. K. T. U. 2010]

(17) Test the consistency of the following system of linear equations and hence find the solution: If exists: (i) $4x_1 - x_2 = 2$, $-x_1 + 5x_2 - 2x_3 = 0$, $-2x_2 + 4x_3 = -8$
 [G.B.T.U. 2006] (ii) $7x_1 + 2x_2 + 3x_3 = 16$, $2x_1 + 11x_2 + 5x_3 = 25$, $x_1 + 3x_2 + 4x_3 = 13$
 [U.B.T.U. 2008]

(18) Investigate for what values of λ and μ do the system of equations $x + y + z = 6$,
 $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ (i) no solution (ii) unique solution (iii) Infinite Solution.
 [G. B. T. U. SUM 2007]

(19) Solve the system of equation $2x_1 + 3x_2 + x_3 = 9$, $x_1 + 2x_2 + 3x_3 = 6$, $3x_1 + x_2 + 2x_3 = 8$ by Gaussian

elimination method: [GBTU 2007]

(20) Show that the of equations $2x + 6y + 11 = 0$, $6x + 20y - 6z + 3 = 0$ and

$6y - 18z + 1 = 0$ are not Consistent. [U.K.T.U. 2011]

(21) Test the consistency of the following system of linear equation and hence find the

solution: $4x_1 - x_2 = 12$, $-x_1 + 5x_2 - 2x_3 = 0$, $-2x_2 + 4x_3 = -8$.

(22) Determine 'b' such that system of homogeneous equation $2x + y + 2z = 0$, $x + y + 3z = 0$, $4x + 3y + bz = 0$ has (i) trivial solution (ii) Non-trivial solution & find Non-trivial Solution using matrix method? : [G.B.T.U. 2009]

(23) Find the value of k so that the equations: $x + y + 3z = 0$, $4x + 3y + kz = 0$,
 $2x + y + 2z = 0$ have a non- trivial solution. [G.B.T.U. (SUM) 2008]

(24) Show that system of equation $3x + 4y + 5z = a$, $4x + 5y + 6z = b$,
 $5x + 6y + 7z = c$ does not have a solution unless $a+c=2b$. [G.B.T.U. (SUM) 2008]

(25) For what values of λ , the equations $x + y + z = 1$, $x + 2y + 4z = \lambda$, $x + 4y + 10z = \lambda^2$
 Have a solution and solve them completely In each case. [G.B.T.U. (C.O.) 2011] (SUM) 2007]

(26) Show that the matrix $\begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$ is Unitary if $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$

[G.B.T.U 2011, 2006]

(27) If $N = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$, Obtain the matrix $(I - N)(I + N)^{-1}$, show that it is Unitary.
[GBTU.CO11]

(28) Express the matrix $A = \begin{bmatrix} i & 2-3i & 4+5i \\ 6+i & 0 & 4-5i \\ -i & 2-i & 2+i \end{bmatrix}$, as a sum of Hermitian and Skew-Hermitian Matrix.
[G.B.T.U. 2010]

(29) Find Eigen values of matrix $A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 0 \\ 3 & 1 & -3 \end{bmatrix}$
[G.B.T.U. 2008]

(30) Find Eigen values of matrix $A = \begin{bmatrix} 2 & 5 & 7 \\ 5 & 3 & 1 \\ 7 & 0 & 2 \end{bmatrix}$
[G.B.T.U. (SUM) 2008]

(31) Find Eigen values and Eigen vectors of the matrix: $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ [GBTU(SUM) 2010]

(32) Find Eigen values and Eigen vectors of the matrix: $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ [G. B. T. U. 2011]

(33) Find Eigen values and Eigen vectors of the matrix: $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ [G.B.T.U. 2009]

(34) Verify Cayley -Hamilton theorem for the matrix: $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$, Find A^{-1} . [GBTU 2006]

(35) Verify Cayley -Hamilton theorem for the matrices (i) $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ [G. B. T. U. 2009]

(ii) $A = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ [G.B.T.U. 2007], (iii) $A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 5 & 4 \\ -4 & 4 & 3 \end{bmatrix}$ [G.B.T.U.] (C.O.) 2010]

(36) Reduce the Matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix}$ to the Diagonalizes ($P^{-1}AP$) form. [G.B.T.U. 2006]

(37) Find a Matrix P which diagonalizes the Matrix $= \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$. Verify $P^{-1}AP = D$, Where D is the Diagonal Matrix.
[G.B.T.U. 2009]

(38) Show that the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable. Hence, find P such that $P^{-1}AP$ is a diagonal matrix. .
[G.B.T.U. C.O.) 2009]

(39) Diagonalize the Matrix (i) $\begin{bmatrix} 3 & 5 \\ 7 & 31 \end{bmatrix}$ [G.B.T.U. (SUM)2009] (ii) $\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ [G.B.T.U. 2007]

(40) Find Eigen values and corresponding Eigen vectors of the matrix and hence diagonalize it.

a. $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ [G.B.T.U. C.O.) 2011]

1. Define velocity and Acceleration in vector calculus. [G.B.T.U. (SUM 2010)]
2. Define Gradient of a scalar field. [G.B.T.U.20006,2007 (SUM 2008)]
3. Find $\text{grad}\phi$ when ϕ is given by $\phi = 3x^2y - y^3z^2$ at the point $(1, -2, -1)$. [G.B.T.U. 2007]
4. Show that $\nabla r^n = nr^{n-2} \vec{r}$ and hence evaluate $\nabla \frac{1}{r}$ where $\vec{r} = xi + yj + zk$.
5. Show that $\nabla(\vec{a} \cdot \vec{r}) = \vec{a}$ where $\vec{r} = xi + yj + zk$ and \vec{a} is a constant vector. [G.B.T.U. 2008]
6. Find a unit Normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point $(1, 2, -1)$. [G.B.T.U. 2014]
7. Find a unit vector Normal to the surface $x^2y + 2xyz = 4$ at the point $(2, -2, 3)$. [G.B.T.U. 2014]
8. What is the greatest rate of increase of $u = xyz^2$ at the point $(1, 0, 3)$?
9. If $\nabla\phi = (y^2 - 2xyz^3)i + (3 + 2xy - x^3y^3)j + (6z^3 - 3x^2yz^2)k$, find k . [G.B.T.U. 2015]
10. Find the Angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.
11. If $u = x + y + z$, $v = x^2 + y^2 + z^2$, $w = yz + zx + xy$, prove that $\text{grad } u, \text{grad } v, \text{and } \text{grad } w$ are Coplanar vectors. [G.B.T.U. 2010, 13, 2015]
12. Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P(1,2,3)$ In the direction of the PQ where Q is the point $(5, 0, 4)$. [G.B.T.U. AG (SUM 2010)]
13. Find the directional derivative of $\phi = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$ at the point $P(3,1,2)$ in the direction Of the vector $yz i + zx j + xy k$. [G.B.T.U. (SUM 2007, 2014)]
14. Find the directional derivative of $\phi = 5x^2y - 5y^2z + \frac{5}{2}z^2x$ at the point $P(1,1,1)$
In the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$. [G.B.T.U. (SUM 2010)]
15. Find a unit vector normal to the surface $xy^3z^2 = 4$ at the point $(-1, -1, 2)$. [G.B.T.U. (SUM 2008)]
16. Find a unit normal vector \hat{n} of the cone of revolution $z^2 = 4(x^2 + y^2)$ at $P(1,0,2)$ [GBTU(CO)2010]
17. If $\phi = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the normal to the surface $x \log z - y^2 + 4 = 0$ at $(2, -1, 1)$. [G.B.T.U. 2011]
18. If $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at point $P(2,1,3)$ in the direction of vector $\vec{a} = i - 2k$. [GBTU(CO)09]
19. If $\vec{r} = xi + yj + zk$ then show that (a) $\square \text{grad } r = \frac{\vec{r}}{r}$ [G.B.T.U. (CO)2011]
(b) $\square \text{grad} \frac{1}{r} = -\frac{\vec{r}}{r^3}$ [G.B.T.U. (CO) 2011] (c) $\square \text{grad} r^n = nr^{n-2} \vec{r}$ [G.B.T.U. 2008 (CO)2011]
(e) $\text{curl}(r^n \vec{r}) = 0$ [G.B.T.U (CO)2010]
20. Prove that $\nabla \log r = \frac{\vec{r}}{r^2}$ [G.B.T.U. (CO) 2011] and $\nabla f(r) = f'(r) \nabla r$ [G.B.T.U. (CO)2011]
21. Find the directional derivative of $\frac{1}{r}$ in the direction \vec{r} where $\vec{r} = xi + yj + zk$ [G.B.T.U. (CO)2012]
22. Find the directional derivative of $\frac{1}{r^2}$ in the direction \vec{r} where $\vec{r} = xi + yj + zk$.
23. Define Divergence and curl of a vector point function. [G.B.T.U.2006, 2007 (SUM) 2008, 2012]
24. If $\vec{r} = xi + yj + zk$, show that (i) $\text{div} \vec{r} = 3$ [G.B.T.U. 2014] (ii) $\text{curl} \vec{r} = 0$ [G.B.T.U. 2014]
25. If $\vec{F}(x, y, z) = xz^3i - 2x^2yzj + 2yz^4k$, Find divergence and curl of $\vec{F}(x, y, z)$. [G.B.T.U. 2007]
26. Find divergence and curl of the vector field $\vec{V} = x^2y^2i + 2xyj + (y^2 - xy)k$. [GBTU(SUM 2007)]
27. A vector field is given by $\vec{A} = (x^2 + xy^2)i + (y^2 + x^2y)j$. Show that the field is irrotational and Find the scalar potential.
28. A fluid motion is given by $\vec{V} = (y + z)i + (z + x)j + (x + y)k$
(i) Is this motion irrotational? If so, find the velocity potential. [A.K.T.U 2016]
(ii) Is the motion possible for an incompressible fluid?

29. Find divergence and curl of the vector field (i) $\vec{V} = xyz\mathbf{i} + 3x^2y\mathbf{j} + (xz^2 - y^2z)\mathbf{k}$
 And $\vec{R} = (x^2 + yz)\mathbf{i} + (y^2 + zx)\mathbf{j} + (z^2 + xy)\mathbf{k}$ [G.B.T.U. 2015]
30. Prove that $(y^2 - z^2 + 3yz - 2x)\mathbf{i} + (3xz + 2xy)\mathbf{j} + (3xy - 2xz + 2z)\mathbf{k}$ is both solenoidal and irrotational. [G.B.T.U. 2009]
31. A fluid motion is given by $\vec{v} = (y \sin z - \sin x)\mathbf{i} + (x \sin z + 2yz)\mathbf{j} + (xy \cos z + y^2)\mathbf{k}$. Is the motion Irrotational? If so find the velocity potential. [G.B.T.U. 2011]
32. If $\vec{F} = (x + y + z)\mathbf{i} + \mathbf{j} - (x + y)\mathbf{k}$, show that $\vec{F} \cdot \text{curl } \vec{F} = 0$.
33. If $\vec{A} = (3xyz^2)\mathbf{i} - (yz)\mathbf{j} + (x + 2z)\mathbf{k}$, find $\text{curl } (\text{curl } \vec{A})$.
34. Show that the vector field $\vec{V} = (\sin y + z)\mathbf{i} + (x \cos y - z)\mathbf{j} + (x - y)\mathbf{k}$ is irrotational.
35. Find the directional derivative of $\nabla \cdot (\nabla \phi)$ at the point $(1, -2, 1)$ in the direction of the normal to the Surface $xyz = 3x + z^2$, where $\phi = 2x^3y^2z^4$. [G.B.T.U. 2009, 2014]
36. Show that $\vec{A} = (6xy + z^3)\mathbf{i} + (3x^2 - z)\mathbf{j} + (3xz^2 - y)\mathbf{k}$ is irrotational. Find the velocity potential ϕ Such that $\vec{A} = \nabla \phi$. [G.B.T.U. 2011, 2014]
37. Find the total work done by the force $\vec{F} = (x^2 + y^2)\mathbf{i} - 2xy\mathbf{j}$ in moving a point from $(0, 0)$ to (a, b) Along the rectangle bounded by lines $x = 0$, $x = a$, $y = 0$, and $y = b$. [G.B.T.U. 2014]
38. Evaluate $\int_S \vec{A} \cdot \mathbf{n} \, ds$, where $\vec{A} = (x + y^2)\mathbf{i} - 2xz\mathbf{j} + 2yz\mathbf{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.
39. Evaluate the line integral $\int_C [(x^2 + xy)dx + (x^2 + y^2)dy]$, where C is the square formed by the lines $x = \pm 1$ and $y = \pm 1$.
40. If $\vec{A} = 2xzi - xj + y^2k$, evaluate $\iiint_V \vec{A} \cdot d\mathbf{V}$, where V is the region bounded by the surface $x = 0, y = 0, x = 2, y = 6, z = x^2, z = 4$.
41. Evaluate $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y, z) = e^{xyz}(yzi + xzj + xyk)$ and $\vec{r} = xi + yj + zk$ and c is the boundary of $0 \leq x \leq 1, 0 \leq y \leq 1$, and $z = 1$. Clockwise. [G.B.T.U. (SUM 2008)]
42. If $\vec{A} = (x - y)\mathbf{i} + (x + y)\mathbf{j}$, evaluate $\oint_C \vec{A} \cdot d\vec{r}$ around curve C consisting of $y = x^2$ and $x^2 = y$. [14]
43. Show that the vector field $\vec{F} = (yz)\mathbf{i} + (zx + 1)\mathbf{j} + (xy)\mathbf{k}$ is conservative. Find the scalar potential. Also find the work done by \vec{F} in moving a particle from $(1, 0, 0)$ to $(2, 1, 4)$. [GBTU 2013]
44. State Gauss Divergence Theorem. (OR)
 Define relation between surface integral and volume integral. [GBTU 2006, 2007, (CO) 11, 2012]
45. Find $\int_S \vec{F} \cdot \mathbf{n} \, ds$, where $\vec{F} = (2x + 3z)\mathbf{i} - (xz + y)\mathbf{j} + (y^2 + 2z)\mathbf{k}$ and S is the sphere having centre at $(3, -1, 2)$ and radius 3. [G.B.T.U. 2006]
46. Define STOKES' Theorem. (OR)
 Define relation between line and surface integral. [G.B.T.U. 2007, (SUM) 2008, 2009, 2012]
47. If $\vec{F} = 3yi - xzj + yz^2k$ and S is the surface of the paraboloid $2z = x^2 + y^2$ bounded by $z = 2$
 Show by using Stoke's theorem that $\int_S (\nabla \times \vec{F}) \cdot \mathbf{n} \, dS = -20\pi$ [G.B.T.U. (SUM 2007)]
48. State Green theorem in the plane. [G.B.T.U. 2008, 2012]
49. Apply Green theorem to evaluate $\oint_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$ where C is the boundary of the area enclosed by x- axis and the upper half of circle $x^2 + y^2 = a^2$. [GBTU (SUM) 2010, (CO) 11]
50. Use Green's theorem to evaluate $\oint_C [(x^2 + xy)dx + (x^2 + y^2)dy]$ where C is the square formed by the lines $y = \pm 1, x = \pm 1$. [G.B.T.U. (CO) 2010]

[B.Tech.1st Year] [AG] [ASSIGNMENT UNIT-3]
[Engineering Mathematics- II (KAG-201)] [2019-20]

(1) Use the method of separation of variables to solve the equation,

$$\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u, \text{ Given that } u(x, 0) = 6e^{-3x} \quad [G.B.T.U. 2004,06, (SUM)10,11]$$

(2) Use method of separation of variables to solve the equation, $\frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$ [GBTU2005,09]

(3) Solve by the method of separation of variables, $4 \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 3u$, $u = 3e^{-x} - e^{-5x}$ when $t=0$

(4) Solve the following equation by the method of separation of variables,

$$\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x, \text{ Given that } u=0 \text{ when } t=0 \text{ and } \frac{\partial u}{\partial t} = 0 \text{ when } x=0. \quad [G.B.T.U. 2008]$$

(5) Find the solution of wave equation. [G.B.T.U. 2005]

(6) A string is stretched and fastened to two points l apart. Motion is started by displacing the string

in the form $y = A \sin \frac{\pi x}{l}$ from which it is released at time $t=0$. Show that the displacement of any point at a distance x from one end at time t is given by-

$$y(x, t) = A \sin \frac{\pi x}{l} \cos \frac{\pi ct}{l} \quad [G.B.T.U. 2004,07,09, (SUM)09]$$

(7) A tightly stretched string with fixed end points $x=0$, and $x = l$ is initially in a position given by

$$y = y_0 \sin^3 \frac{\pi x}{l}. \text{ If it is released from rest from this position,}$$

Find the displacement $y(x,t)$. [G.B.T.U. (CO)2011]

(8) If a string of length l is initially at rest in equilibrium position and each of its point is given the

(9) Find the equation of one dimensional heat flow and its solution. [G.B.T.U. (CO)2011]

(10) A rod of length l with insulated sides is initially at a uniform temperature u_0 . Its ends are suddenly cooled to $0^\circ C$ and are kept at the temperature. Find the temperature *function* $u(x,t)$.

[G.B.T.U. (SUM) 2010, 2011]

(11) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary condition, $u(x, 0) = 3 \sin \pi x, u(0, t) = 0$,

$$u(l, t) = 0 \text{ where } 0 < x < l. \quad [G.B.T.U. 2002]$$

(12) A bar with insulated sides is initially at a temperature $0^\circ C$ throughout. The end $x = 0$, is kept

At 0°C , and heat is suddenly applied at the end $x = l$, so that $\frac{\partial u}{\partial x} = A$ for $x = l$, where A is

Constant. Find the temperature function $u(x, t)$. [G.B.T.U. 2002]

(13) Use separation of variables method to solve the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the boundary conditions $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin \frac{n\pi x}{l}$ [G.B.T.U. 2003,04]

(14) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in a rectangular in the xy-plane with $u(x, 0) = 0$, $u(x, b) = 0$, $u(0, y) = 0$ and $u(a, y) = f(y)$ parallel to y- axis. [G.B.T.U. (SUM)2008]

(15) Solve the equation by the method of separation of variables $u_{xx} + u_y + 2u$,

$u(0, y) = 0$, $\frac{\partial}{\partial x} u(0, y) = 1 + e^{-3y}$ [G.B.T.U. 2009,2010]

(16) Solve by the method of separation of variables, $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$ $u(0, y) = 8e^{-3y}$ [G.B.T.U. 2008]

(17) Solve by the method of separation of variables, $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y} = 0$ [G.B.T.U. (SUM) 2007]

(18) Solve by the method of separation of variables $y^3 \frac{\partial u}{\partial x} + x^2 \frac{\partial u}{\partial y} = 0$ [G.B.T.U. 2011]

(19) Show how the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ can be Solve by the method of separation of variables.

(20) Find the deflection $u(x, y, t)$ on the tightly stretched rectangular membrane with sides a and b having wave velocity $c = 1$, if the initial velocity is zero and its deflection is

$$f(x, y) = \sin \frac{2\pi x}{a} \sin \frac{3\pi y}{a} \quad [G.B.T.U. 2011]$$

(21) Find the solution of heat equation. [G.B.T.U. (CO) 2011],2007]

(22) Find the temperature in a bar of length 2, whose ends are kept at zero and lateral surface insulated if the initial temperature $\frac{\pi x}{2} + 3\sin \frac{5\pi x}{2}$. [G.B.T.U. (CO)2007,2009]

(23) Find the solution of Laplace Equation in Two Dimension. [G.B.TU 2011 (CO)10,]

(24) Solve by the method of separation of variables, $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the boundary condition $u(0, y) = u(l, y) = u(x, 0) = 0$, and $u(x, a) = \sin \frac{n\pi x}{l}$. [G.B.TU(CO)2009,]

(25) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in a rectangular with $u(0, y) = 0$, $u(a, y) = 0$, $u(x, b) = 0$ and $u(x, 0) = f(x)$ parallel to x- axis. [G.B.T.U. 2008]

