VISION INSTITUTE OF TECHNOLOGY KANPYUR
SIGNMENTS [MATHS-I] [KAS203]
[UNIT – I] [DIFFERENTIAL EQUATIONS] [2019]

Q.1 Determine the differential equation whose set of independent solutions is $\{e^x, xe^x, x^2e^x\}$. [A.K.T.U.2018]

Q.2 If $y = y_1(x)$ and $y = y_2(x)$ are two solutions of the equation $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0, then$ show that

$$y_1\left(\frac{dy_2}{dx}\right) - y_2\left(\frac{dy_1}{dx}\right) = ce^{-\int P\,dx}, where\ c\ is\ the\ constant.$$ [A.K.T.U.2017]

Q.3 Solve the equation $x\frac{dy}{dx} - \frac{1}{2}y = x + 1$ and prove that the only solution for which x and y can obtain the value unity is given by $y = 2x + \sqrt{x} - 2$. [G.B.T.U.(C.O.) 2011]

Q.4 Solve: $\frac{d^2y}{dx^2}$ +2$\frac{dy}{dx} + 10y + 37sin3x$= 0 and find the value of y when $x = \frac{\pi}{2}$ being given that
$y = 3, \frac{dy}{dx} = 0, when\ x = 0.$ [G.B.T.U. 2011]

Q.5 Solve: $(D^2 - 3D + 2)y = x^2 + 2x + 1$ [A.K.T.U.2015, 2016]

Q.6 Solve: $(D^2 + 5D - 6)y = \sin 3x + \cos 2x$ [G.B.T.U. 2010, CO 2011, CO 2012]

Q.7 Solve: $\frac{d^2y}{dx^2} + 4y = sin^2 2x$, with conditions$y(0) = 0, y'(0) = 0.$ [G.B.T.U. 2013]

Q.8 Solve: $\frac{d^2y}{dx^2}$ + y = e^{2x}+ $\cosh 2x$ + x^3 [G.B.T.U. 2014]

Q9. Solve: $(D^2 - 2D + 1)$ y = $e^x \sin x$ [G.B.T.U. 2016, 2017]

Q.10 Solve: $(D^2 - 2D + 1)$ y = x^2e^{3x} [G.B.T.U 2014]

Q.11 Solve: $(D^2 - 2D + 5)$ y = $e^{2x} \cos x$ [G.B.T.U.2013]

Q.12 Solve: $(D^2 - 2D + 1)y = x\ sinx$ [G.B.T.U. 2012]

Q.13 Solve: $(D^2 - 2D + 4)y = e^x \cos x + sinx\ cos3x.$ [A.K.T.U.2018]

Q.14 Solve: $(D^2 + 2D + 1)$ y = $x^2e^{-x} \cos x$ [G.B.T.U. 2012]

Q.15 Find the complete solution of $(D^2 + a^2)y = sec\ ax.$ [A.K.T.U.2011, 2017]

Q16. Solve: $(D^2 - 4D + 4)$ y = $e^x \cos x$ [G.B.T.U.(CO) 2010]

Q17. find the complete solution of $(D^2 - 3D + 2)$ y = x$e^{3x} + \sin 2x$ [G.B.T.U. (SUM) 2010]

Q.18 Solve: $(D^2 + 2D + 2)$ y = $e^{-x}sec^3x$ [A.K.T.U. 2017]

Q.19 Solve: $\frac{d^2y}{dx^2}$ + 2$\frac{dy}{dx}$ + y = $\frac{e^{-x}}{x+2}$

Q.20 Solve: $(\boldsymbol{x} + \mathbf{1})^2\frac{\boldsymbol{d^2y}}{\boldsymbol{dx^2}} + (x+1)\frac{\boldsymbol{dy}}{\boldsymbol{dx}} = (\mathbf{2}\boldsymbol{x} + \mathbf{3})(\mathbf{2}\boldsymbol{x} + \mathbf{4})$ [**G. B. T. U.** (CO) **2011**]

Q.21 Solve: $x^2\frac{d^2y}{dx^2}$ +x $\frac{dy}{dx} - y = x^3e^x$ [A.K.T.U. 2016]

Q.22 Solve: $x^2\frac{d^2y}{dx^2}$ +x $\frac{dy}{dx}$ +y = $(logx)\sin(\log x)$ [G.B.T.U. 2015]

Q.23 Solve the following simultaneous differential equations
$\frac{dx}{dt}$ =3x+2y and $\frac{dy}{dt}$ =5x+3y [G.B.T.U.2011, 2008]

Q.24 Solve the following simultaneous differential equations
$\frac{dx}{dt}$ +5x-2y=t and $\frac{dy}{dt}$ +2x+y = 0; given that x=y=0 when t=0 [G.B.T.U. 2007, 2008, 2015]

Q.25 Solve the simultaneous differential equations.
$\frac{d^2x}{dt^2} + y = sint \quad and \quad \frac{d^2y}{dt^2} + x = cost$ [A.K.T.U. 2016]

Q.26 Solve: $\frac{dx}{dt}$ +4x+3y=t and $\frac{dy}{dt}$ +2x+5y = e^t [G.B.T.U. 2006, 2015]

Q.27 Solve the simultaneous differential equations.
$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 4x = y \quad and \quad \frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = 25x + 16e^t$ [G.B.T.U. 2016]

Q.28 Solve: $\frac{dx}{dt}$ +7x-y=0 and $\frac{dy}{dt}$ +2x+5y = 0 [G.B.T.U. 2006, 2015]

Q.29 Solve: $\frac{\boldsymbol{dx}}{\boldsymbol{dt}} = -4(x+y), \frac{\boldsymbol{dx}}{\boldsymbol{dt}} + 4\frac{\boldsymbol{dy}}{\boldsymbol{dt}} = -4y\ with\ cond.\ x(0) = 1, y(0) = 0$ [GBTU 2011,2014]

Q.30 Solve: $\frac{dx}{dt} + y = \sin t$ and $\frac{dy}{dt} + x = \cos t$ [G.B.T.U. (CO) 2012]

Q.31 Solve: $\frac{dx}{dt} + x - 2y = 0,:$ $\frac{dy}{dt} + \mathrm{x} + 4\mathrm{y} = 0,:$ $\mathrm{x}(0) = \mathrm{y}(0) = 0$ [G.B.T.U (2015]

Q.32Solve $\frac{dx}{dt} - y = e^t : \frac{dy}{dt} + \mathrm{x} = \mathrm{sint}; \mathrm{x}(0) = 1, \mathrm{y}(0) = 0$ [G.B.T.U (SUM). 2010, (CO) 2011]

Q.33 Solve $\frac{dx}{dt}+\frac{dy}{dt}+2x+y=0,\ \ \frac{dy}{dt}+5x+3y=0$ [G.B.T.U (CO). 2011]

Q.34 Solve: $\frac{dx}{dt}+2x+4y=1+4t\ \ and\ \ \frac{dy}{dt}+x-y=\frac{3}{2}t^2.$ [G.B.T.U. 2013]

Q.35 Solve: $\frac{d^2y}{dx^2}-2tanx\frac{dy}{dx}+5\,y=e^xSecx$ [G.B.T.U. 2015]

Q.36 Solve: $\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+(x^2-8)\,y=x^2e^{-\frac{x^2}{2}}$. [G.B.T.U 2012,2013]

Q.37 By changing the independent variables, solve the differential equation:

$\frac{d^2y}{dx^2}-\frac{1}{x}\frac{dy}{dx}+4x^2y=x^4$ [G.B.T.U.2001, 03, 04, 2015]

Q.38 By changing the independent variables, solve the differential equation:

$\frac{d^2y}{dx^2}+(3sinx-cotx)\frac{dy}{dx}+2y\,sin^2x=e^{-cosx\,sin^2x}$ [G.B.T.U. 2015]

Q.39 Solve: $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dt}+\ y=4cos\,log(1+x).$

Q.40 Solve: By changing the independent variables,

$\frac{d^2y}{dx^2}-cotx\frac{dy}{dx}-y\,sin^2x=cosx-\ Cos^3x$ [G. B. T. U. 2011, 2013]

Q.41 Solve: By changing the independent variables,

$x^6\frac{d^2y}{dx^2}+3x^5\frac{dy}{dx}+a^2y=\frac{1}{x^2}$ [G. B. T. U. 2014]

Q.42 Solve: by the method of variation of parameters,

$\frac{d^2y}{dx^2}+a^2y=secax$ [G.B.T.U.2005, 08, 13, 14, 2015]

Q.43 Solve: by the method of variation of parameters, $\frac{d^2y}{dx^2}+y=tanx$ [G. B. T. U. 2011,2015]

Q.44 Use the variation of parameters method to solve the differential equation: $x^2y''+xy'-y=x^2e^x$ [G. B. T. U. 2004,06, 2018]

Q.45 Solve: by the method of variation of parameters,

$x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-12y=0$ [A. K. T. U. 2016]

Q.46 Solve: by the method of variation of parameters,

$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=\frac{e^x}{1+e^x}$

Q:47 Solve the differential equation $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^2+2logx$ [G.B.T.U (CO) 2012]

Q.48 Use the variation of parameters method to solve the differential equation: $x^2y''+4xy'+2y=e^x$ [G. B. T. U. 2012]

Q.49 Use the variation of parameters method to solve the differential equation,

$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-9y=48x^5$ [G. B. T. U. 2004, (CO) 2010]

Q.50 Use the variation of parameters method to solve the differential equation,

$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=\frac{e^{3x}}{x^2}$ [G. B. T. U. 2017]

[ALL THE BEST]

ASSIGNMENTS [MATHS-II] [KAS203]

[UNIT – II] [SEQUENCES AND SERIES] [2019]

1. Define Bounded and unbounded sequences with examples.
2. Define convergent, divergent and oscillating sequences with examples.
3. Define Monotonic sequences.
4. Show that every convergent sequence is bounded but converse is always not true.
5. Discuss the convergence of the following sequences $\{a_n\}$ where :

   $(i)\ a_n=\frac{n+1}{n}$ $(ii)\ a_n=\frac{n}{n^2+1}$ $(iii)\ a_n=1+\frac{1}{3}+\frac{1}{3^2}+\ \dots\dots\dots+\frac{1}{3^n}.$
6. Define Limit U_n Test.

7. Define Leibnitz's test (OR) Alternating series test with examples.

8. Examine the series: $2+\frac{3}{2}+\frac{4}{3}+\frac{5}{4}+\cdots \ldots \ldots \ldots \ldots$

9. Test the series: $\sqrt{\frac{2}{3}}+\sqrt{\frac{3}{4}}+\sqrt{\frac{4}{5}}+\cdots \ldots \ldots \ldots \ldots \ldots \ldots$

10. Test whether the series: $\frac{1}{1+2^{-1}}+\frac{2}{1+2^{-2}}+\frac{3}{1+2^{-3}}+\cdots \ldots$ is convergent or divergent.

11. Test the convergence of the series : $\sqrt{\frac{1}{2}}+\frac{2}{\sqrt{5}}+\frac{4}{\sqrt{17}}+\frac{8}{\sqrt{65}}+\cdots \ldots \ldots \ldots \ldots .+\frac{2^n}{\sqrt{4^n+1}}+\cdots \ldots$

12. Prove that the series : $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots \ldots$ is convergent.

13. Test convergence of the series(i) $2-\frac{3}{2}+\frac{4}{3}-\frac{5}{4}+\frac{6}{5}-\cdots$ (ii) $\log\frac{1}{2}-\log\frac{2}{3}+\log\frac{3}{4}-\log\frac{4}{5}+..$

14. ii) $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots \ldots$ (iii) $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+\cdots \ldots$ (iv) $1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+\cdots \ldots$

15. Define Comparison test for positive term series.

16. State $p-Test$ (OR) Hyper-Harmonic test.

17. Test the series : $1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\cdots \ldots$

18. Test the series : (i) $\frac{14}{1^3}+\frac{24}{2^3}+\frac{34}{3^3}+\cdots \ldots .+\frac{10n+4}{n^3}+\cdots \ldots \ldots$ (ii) $\frac{\sqrt{1}}{1+\sqrt{1}}+\frac{\sqrt{2}}{1+\sqrt{2}}+\frac{\sqrt{3}}{1+\sqrt{3}}+\cdots .$

(iii) $1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+\cdots$ (iv) $\frac{2^p}{1^q}+\frac{3^p}{2^q}+\frac{4^p}{3^q}+\cdots \ldots$(v) $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\cdots ..$ (vi) $\sqrt{n^3+1}-\sqrt{n^3}$

(vii) $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+\cdots \ldots$ (viii) $\frac{1}{\sqrt{2}-1}+\frac{1}{\sqrt{3}-1}+\frac{1}{\sqrt{4}-1}+\cdots \ldots$ (ix) $\frac{1}{5}+\frac{\sqrt{2}}{7}+\frac{\sqrt{3}}{9}+\frac{\sqrt{4}}{11}+\cdots \ldots$

(x) $1+\frac{1+2}{1+2^2}+\frac{1+3}{1+3^2}+\cdots$ (xi) $\frac{\sqrt{3}}{1.2}+\frac{\sqrt{5}}{3.4}+\frac{\sqrt{7}}{5.6}+\frac{\sqrt{9}}{7.8}+\cdots$

19. Test for convergence or divergence of the series whose n^{th} term (general term) are:-

(i) $\frac{1}{na+b}$ (ii) $\frac{2n+1}{n(n+1)(n+2)}$ (iii) $\sqrt{n+1}-\sqrt{n}$ (iv) $\sqrt{n^2+1}-n$ (v) $\sqrt{n+1}-\sqrt{n-1}$

(vi) $\sqrt{n^2+1}-\sqrt{n^2-1}$ (vii) $\sqrt{n^3+1}-\sqrt{n^3-1}$ (vii) $(n^3+1)^{\frac{1}{3}}-n$ (viii) $\frac{1}{\sqrt{n}+\sqrt{n-1}}$

(ix) $\frac{n}{1+n\sqrt{n+1}}$ (x) $\sin\frac{1}{n}$ (xi) $\cos\frac{1}{n}$ (xii) $\frac{1}{n}\sin\frac{1}{n}$ (xiii)$\tan^{-1}\frac{1}{n}$

20. State Cauchy's Root Test (OR) Radical Test.

21. Test the series: (i) $\sum(1-\frac{1}{n})^{n^2}$ (ii) $(\frac{2^2}{1^2}-\frac{2}{1})^{-1}+(\frac{3^3}{2^3}-\frac{3}{2})^{-2}+(\frac{4^4}{3^4}-\frac{4}{3})^{-3}+\cdots \ldots \ldots$

(iii) $(1+\frac{1}{n})^{n^2}$ (iv) $(1+\frac{1}{n})^{-n^2}$ (v) $[\log(1+\frac{1}{n})]^n$ (vi) $(1+\frac{1}{\sqrt{n}})^{-n^{\frac{3}{2}}}$ (vii) $\sum_{n=2}^{\infty}(\frac{1}{\log n})^n$

22. State D'Alembert's Test (OR) Ratio Test.

23. Test the series: (i) $\frac{2}{1^2+1}+\frac{2^2}{2^2+1}+\frac{2^3}{3^2+1}+\cdots \ldots \ldots$ (ii) $1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+\cdots \ldots+\frac{x^n}{n^2+1}+\cdots \ldots$

(iii) $\frac{1}{1.2.3}+\frac{x}{4.5.6}+\frac{x^2}{7.8.9}+\cdots ..$ (iv) $\frac{1}{1+2}+\frac{2}{1+2^2}+\frac{3}{1+2^3}+\cdots .$ (v) $\frac{1!}{5}+\frac{2!}{5^2}+\frac{3!}{5^3}+\cdots \ldots$

(vi) $1+\frac{2^2}{2!}+\frac{3^2}{3!}+\frac{4^2}{4!}+\cdots ..$ (vii) $1+2x+3x^2+4x^3+\cdots .$ (viii) $1^2+2^2x+3^2x^2+4^2x^3+\cdots \ldots ..$

(ix) $\sum\frac{3n-1}{2^n}$ (x) $\sum\frac{x^n}{n(n+1)}$ (xi) $\sum\frac{x^n}{a+\sqrt{n}}$ (xii) $\sum\frac{1}{x^n+x^{-n}}$

24. Define Raabe's and Logarithmic Test.

25. Test the convergence of the series: $1+\frac{2}{3}.\frac{1}{4}+\frac{2}{3}.\frac{4}{5}.\frac{1}{6}+\frac{2}{3}.\frac{4}{5}.\frac{6}{7}.\frac{1}{8}+\cdots \ldots \ldots \ldots \ldots \ldots \ldots$

26. Test the convergence of the series: $1+\frac{x}{2}+\frac{1}{2}.\frac{3}{4}.x^2+\frac{1}{2}.\frac{3}{4}.\frac{5}{6}x^3+\cdots \ldots \ldots \ldots .,x>0.$

27. Test the series: $x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+\frac{5^5x^5}{5!}+\cdots\ldots\ldots\ldots$

28. Test the series: $1+\frac{2^2}{3.4}+\frac{2^2.\ 4^2}{3.4.5.6}+\frac{2^2.4^2.6^2}{3.4.5.6.7.8}+\cdots\ldots\ldots\ldots\ldots\ldots$

29. Test the series: $\sum\frac{1.2.3\ldots\ldots n}{4.7.10\ldots\ldots\ldots(3n+1)}x^n$ (30) $xlogx+x^2log2x+x^3log3x+\cdots..+x^n\ lognx+\cdots$

[FOURIER SERIES]

Q.1 Define Periodic Function.

Q.2Obtain the Fourier series to represent f(x) = $\frac{1}{4}(\pi-x)^2$in the interval (0, 2π) and hence obtain the following relations.

(a) $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots\ldots=\frac{\pi^2}{6}$ (b) $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots=\frac{\pi^2}{12}$ (c)) $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots.=\frac{\pi^2}{8}$ [G.B.T.U.2008]

Q.3 Obtain the Fourier series for f(x) = e^{-x} in the interval 0<x<2π. [G.B.T.U.2001, (SUM) 2010]

Q.4 Expand f(x) = x$\sin x$, 0<x<2π as a Fourier series. [G.B.T.U.2001,(SUM) 2010]

Q.5 Find the Fourier series for the function f(x) = x+ x^2 , -π<x<π and hence [G.B.T.U.2003, (SUM) 2010]

Show that (i) $\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\ldots.$ (ii) $\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots.$

Q.6 Express f(x) =$|x|$, -π<x<π as a Fourier series, hence show that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots.=\frac{\pi^2}{8}$ [G.B.T.U.2001]

Q.7 Obtain the Fourier series for the function f(x) =x^2, -π<x<π.sketch the graph and hence show that

(a) $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots\ldots=\sum\frac{1}{n^2}=\frac{\pi^2}{6}$ [G.B.T.U.2001]

(b) $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots=\frac{\pi^2}{12}$ [G.B.T.U.2004]

Q.8 Expand the function f(x) = x $\sin x$ as a Fourier series in the interval -$\pi\le x\le\pi$.

Deduce that $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\frac{1}{7.9}+\ldots\ldots\ldots\ldots=\frac{\pi-2}{4}$ [G.B.T.U.2001, 2005, 2008]

Q.9 Expand in a Fourier series the function f(x) = x in the interval 0<x<2π. [G.B.T.U.2001]

Q.10 Expand f(x) = $|\cos x|$as a Fourier series in the interval -π<x<π [G.B.T.U.2004]

Q.11 Obtain a Fourier series to represent f(x) = x^2 in the interval (0, 2π) and hence [G.B.T.U.2008]

Deduce that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots=\frac{\pi^2}{12}$

(12) Obtain Fourier series for the function $f(x)=\begin{cases}x, & -\pi<x<0\\ -x, & 0<x<\pi\end{cases}$ and hence

Show that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\ldots\ldots\ldots\ldots=\frac{\pi^2}{8}$ $[G.B.T.U.\ 2002,(CO)2010,(SUM)10]$

(13) Obtain Fourier series for the function $f(x)=\begin{cases}x, & -\pi<x<0\\ -x, & 0<x<\pi\end{cases}$ and hence

Deduce that $\frac{1}{1}-\frac{1}{4}+\frac{1}{9}-\frac{1}{16}+\ldots\ldots=\frac{\pi^2}{12}$ $[G.B.T.U.(CO)2012]$

(14) Find the Fourier series for the function defined by $f(x)=\begin{cases}-1, & -\pi<x<0\\ 0, & x=0\\ 1, & 0<x<\pi\end{cases}$

Hence prove that $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}\ldots\ldots\ldots$ $[G.B.T.U.\ 2005,2012]$

(15) Obtain the Fourier series expansion of $f(x)=\left(\frac{\pi-x}{2}\right)$ for $0<x<2$ $[G.B.T.U.\ 2005]$

(16) Find Fourier expansion for the following $f(x)=x-x^2,\quad -1<x<1$ $[G.B.T.U.\ 2005]$

(17) Obtain Fourier series for function $f(x)=\begin{cases}\pi x, & 0\le x\le 1\\ \pi(2-x), & 1\le x\le 2\end{cases}$ $[G.B.T.U.\ 2001,07]$

(18) Expand $\pi x-x^2$ in a half range Sine series in the interval (0, π) upto the First three terms. $[G.B.T.U.\ 2001]$

(19) Find a series of cosine of multipliers of x which will represent xsinx in the interval (0, π)

And show that:- $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\cdots\ldots\ldots=\frac{\pi-2}{4}$ $[G.B.T.U.\ 2002]$

(20) Expand $f(x)=x$ as a half range

(i) Sine series in 0$< x<2$ $[G.B.T.U.\ 2001,07]$

(ii) Cosine series in 0$< x<2$ $[G.B.T.U.\ 2004,07]$

(21) Obtain the half-range sine series for $f(x)=x-x^2$ is the interval 0$< x<1$ $[GBTU\ 2012]$

(22) Define even and odd function. $\left[G.B.T.U.\ 2009,[G.B.T.U.\ 2009]\right]$

(23) Obtain the Fourier series expansion of $f(x)=\left(\frac{\pi-x}{2}\right)$ for $0<x<2\pi$. Deduce that .

$\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}\ldots\ldots\ldots..$ $[G.B.T.U.2007,09,\ \ M.T.U.2011]$

(24) Find Fourier expansion for the following $f(x)=x^3$ in $(-\pi<x<\pi)$. $[G.B.T.U.\ (SUM)2009]$

(25) Obtain Fourier series for the function $f(x)=\begin{cases}x, & 0\le x\le\pi\\ 2\pi-x, & \pi\le x\le\pi\end{cases}$ and hence

Show that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\ldots\ldots\ldots\ldots=\frac{\pi^2}{8}$ $[G.B.T.U.(SUM)2010]$

(26) Find Fourier series for following periodic function, $f(x)=\begin{cases}-k, & -\pi<x<0\\ k\,, & 0<x<\pi\end{cases}$

Also prove that: $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}\ldots\ldots\ldots..$ $[G.B.T.U.\ 2009,\ 2010]$

(27) Find Fourier series for periodic function, $f(x)=\begin{cases}x^2, & -\pi<x<0\\ -x^2, & 0<x<\pi\end{cases}$

(MATH-II) UNIT- (KAS-203) (ASSIGNMENT)

(1) Define Bounded function with examples.

(2) Examine the convergence of the improper integrales:

(a) $\int_1^\infty \frac{1}{x}\, dx$ (b) $\int_1^\infty \frac{dx}{\sqrt{x}}$ (c) $\int_1^\infty \frac{dx}{x^{\frac{3}{2}}}$ (d) $\int_0^\infty \frac{1}{1+x^2}\, dx$. (e) $\int_a^\infty \frac{x}{1+x^2}\, dx$

(f) $\int_0^\infty \frac{1}{(1+x)^3}\, dx$ (g) $\int_0^\infty \frac{1}{x^2+4a^2}\, dx$ (h) (e) $\int_3^\infty \frac{1}{(x-2)^2}\, dx$ (f) (e) $\int_{\sqrt{2}}^\infty \frac{1}{x\sqrt{x^2-1}}\, dx$

(3) Examine the convergence the integrales: (a) $\int_1^\infty xe^{-x}dx$ (b) $\int_0^\infty x^2e^{-x}dx$

(c) $\int_0^\infty xe^{-x^2}dx$ (d) $\int_0^\infty x^3e^{-x^2}dx$ (e) $\int_0^\infty x\, sinx\, dx$

(4) Examine the convergence the integrales:

(a) $\int_1^\infty \frac{dx}{(1+x)\sqrt{x}}$ (b) $\int_2^\infty \frac{dx}{x\, logx}$ (c) $\int_0^\infty e^{-x}\, sinx\, dx$ (d) $\int_0^\infty e^{-ax}\, cosbx\, dx$

(5) Examine the convergence the integrales:

(a) $\int_1^\infty \frac{dx}{x(1+x)}$ (b) $\int_1^\infty \frac{dx}{x^2(x+1)}$ (c) $\int_1^\infty \frac{tan^{-1}x}{x^2}\, dx$ (d) $\int_0^\infty e^{-\sqrt{x}}\, dx$

(6) Examine the convergence the integrales:

(a) $\int_{-\infty}^0 e^{2x}\, dx$ (b) $\int_{-\infty}^0 \frac{dx}{p^2+q^2x^2}$ (c) $\int_{-\infty}^0 e^{-x}\, dx$ (d) $\int_{-\infty}^0 sinhx\, dx$

(7) Examine the convergence the integrales:

(a) $\int_{-\infty}^\infty e^{-x}\, dx$ (b) $\int_{-\infty}^\infty \frac{1}{1+x^2}\, dx$ (c) $\int_{-\infty}^\infty \frac{1}{e^x+e^{-x}}\, dx$

(8) Test the convergence the integrales: (a) $\int_0^1 \frac{dx}{\sqrt{x}}$ (b) $\int_0^1 \frac{dx}{x^2}$ (c) $\int_1^2 \frac{x}{\sqrt{x-1}}\, dx$

(9) Examine the convergence the integrales:

(a) $\int_0^1 logx\, dx$ (b) $\int_0^{\frac{1}{e}} \frac{1}{x(logx)^2}\, dx$ (c) $\int_1^2 \frac{1}{x\, logx}\, dx$

(10) Examine the convergence the integrales:

(a) $\int_0^a \frac{1}{\sqrt{a-x}}\, dx$ (b) $\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$ (c) $\int_0^{\frac{\pi}{2}} \frac{cosx}{\sqrt{1-sinx}}\, dx$

(11) Examine the convergence the integrales:

(a) $\int_{-1}^1 \frac{1}{x^2}\, dx$ (b) $\int_a^{3a} \frac{1}{(x-2a)^2}\, dx$

(12) Examine the convergence the integrales:

(a) $\int_0^4 \frac{1}{x(4-x)}\, dx$ (b) $\int_{-a}^a \frac{x}{\sqrt{a^2-x^2}}\, dx$ (c) $\int_0^\pi \frac{1}{1+cosx}\, dx$ (d) c) $\int_0^\pi \frac{1}{sinx}\, dx$

(13)(i) The improper integral $\int_a^b \frac{dx}{(x-a)^n}$ is convergent if and only if $n < 1$.

(ii) The improper integral $\int_a^b \frac{dx}{(b-x)^n}$ is convergent if and only if $n < 1$.

(14) The improper integral $\int_a^\infty \frac{1}{x^n}\, dx$, $(a > 0)$ is convergent if and only if $n > 1$.

(15) Define Beta and Gamma function. [G.B.T.U. 06, 08, (SUM) 08 , 2018]

(16) Prove that $\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$ [G.B.T.U. 10, G.B.T.U(SUM) 09 , AKTU2018]

(17) Prove that: $\Gamma(m)\Gamma\left(m+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{(2)^{2m-1}}\Gamma(2m)$, where m is positive integer. [G.B.T.U. 2013]

(OR) State and prove Duplication formula.

(18) Using Beta and Gamma functions, evaluate: (i) $\int_0^\infty x^{\frac{1}{4}}\, e^{-\sqrt{x}}dx$. [G.B.T.U. (SUM) 2008]

(ii) $\int_0^1 \left(\frac{x^3}{1-x^3}\right)^{\frac{1}{2}} dx$. [G.B.T.U. 07,14,18] (iii) $\int_0^\infty \frac{dx}{1+x^4}$ [G.B.T.U. 2012]

(19) Prove that :- $\beta(l,m).\beta(l+m,n).\beta(l+m+n,p) = \frac{\Gamma(l)\Gamma(m)\,\Gamma(n)\Gamma(p)}{\Gamma(l+m+n+p)}$ [G.B.T.U. 2008]

(20) Evaluate : $\int_0^1 \frac{1}{\sqrt{1+x^4}}\, dx$ [G.B.T.U. 2011]

(21) Prove that :− $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$ [G.B.T.U. 2009]

(22) Evaluate the integral $\int\int\int x^{l-1} y^{m-1} z^{n-1}\ dx\, dy\, dz$ where x,y,z are all positive but limit By the condition $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \le 1.$ [G.B.T.U. (SUM) 2009, G.B.T.U. 2006, 2011]

(23) Apply Dirichlet's integral to find the mass of an octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, The density at any point being $\rho = kxyz.$ [G.B.T.U. 2006, 2007, (C.O.)2011, 2015]

(24) Prove that : $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{sin\theta}} = \int_0^{\frac{\pi}{2}} \sqrt{sin\theta}\, d\theta = \pi.$ [G.B.T.U. 2014]

(25) Show that the area bounded by the curve $x^n + y^n = a^n$ and the co-ordinate axis in the First quadrant $\frac{a^2\, \Gamma\left(\frac{1}{n}\right)^2}{2n\, \Gamma\left(\frac{2}{n}\right)}$. [G.B.T.U. (C.O) 2011]

(26)Find Area and mass contained in the first quadrant enclosed by the curve $\left(\frac{x}{a}\right)^\alpha + \left(\frac{y}{b}\right)^\beta = 1$ Where $\alpha > 0, \beta > 0$ given that density at any point$\rho(x,y)$ *is* $k\sqrt{xy}.$ [G.B.T.U. 2009]

(27) Prove that: $\int_0^\infty \sqrt{y}\ e^{-y^2}\ dy \times \int_0^\infty \frac{e^{-y^2}}{\sqrt{y}}\ dy = \frac{\pi}{2\sqrt{2}}.$ [G.B.T.U. (C.O) 2011]

(28) Prove that: $\int_0^1 \frac{x^2\, dx}{\sqrt{1-x^4}} \times \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}.$ [G.B.T.U. (SUM) 2008]

(29) Prove that: $\int_0^{\frac{\pi}{2}} \sqrt{tan\theta}\ d\theta = \int_0^{\frac{\pi}{2}} \sqrt{cot\theta}\ d\theta = \frac{\pi}{\sqrt{2}}.$ [G.B.T.U. 2013]

(30) Find the volume contained in the solid region in the first octant of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$ [G.B.T.U. 2014]

(31) Evaluate $\int\int\int_V e^{-(x+y+z)}\ dx\, dy\, dz$, where the region of integration is bounded by Planes $x = 0, y = 0, z = 0.$ and $x + y + z = a.\ a > 0.$ [G.B.T.U. (SUM) 2008]

(32) Evaluate $\Gamma\left(\frac{1}{4}\right) \Gamma\left(\frac{3}{4}\right)$. [G.B.T.U. (CO) 2013]

(33) Prove that $\iiint \frac{dx\, dy\, dz}{\sqrt{1-x^2-y^2-z^2}} = \frac{\pi^2}{8}$, the integral being extended to all positive values of the vairiables for which the expression is real. [G.B.T.U.,2014 , 2016]

(34) Evaluate $\int\int\int_R (x^2 + y^2 + z^2)\, dx\, dy\, dz$ where R *denotes the region bounded by* $x = 0, y = 0,\quad z = 0$ *and* $x + y + z = a,\ (a > 0.)$ [G.B.T.U. (CO) 2013]

(35) Find the mass of solid $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r = 1$, the density at any point being $\rho = kx^{l-1}y^{m-1}z^{n-1}$ where x, y, z are all positive. [G.B.T.U. 2016]

(36) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the axis in A, B *and* C . Apply Dirichlet's integral to find the volume of the tetrahedron ABC . Also find its mass if the density at any point is $kxyz$. [12,18]

(37) Evaluate. $\Gamma\left(-\frac{3}{2}\right) = \frac{4}{3}\sqrt{\pi}$ [G.B.T.U. 2013]

(38) $\Gamma\left(-\frac{1}{2}\right) = -2\sqrt{\pi}$ [G.B.T.U. 2010]

(39) Evaluate. $\iiint\ (ax^2 + by^2 + cz^2)\, dx\, dy\, dz$, *Where V is the Region bounded by* $x^2 + y^2 + z^2 \le 1.$ [G.B.T.U. 2013]

(40) Evaluate:- (i) $\Gamma\left(-\frac{5}{2}\right)$ (ii) $\Gamma\left(-\frac{7}{2}\right)$.

(41) Compute $\iiint_V x^2\ dx\, dy\, dz$, over volume of tetrahedron bounded by $x = 0, y = 0, z = 0$ And $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. [G.B.T.U 2017]

(42) Evalute $\iiint_V x^2yz\,dx\,dy\,dz$, throughout the volume bounded by planes

$x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. [G.B.T.U 2017] .

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(43) Evaluate: (i) $\frac{\beta(m+1,n)}{\beta(m,n)}$ [G.B.T.U 2011] (ii) Evaluate: $\int_0^{\infty} e^{-x^2}\,dx$ [G.B.T.U 2011]

(82) Prove that: $\frac{B(p,\ q+1)}{q} = \frac{B(p+1,\ q)}{p} = \frac{B(p,\ q)}{p+q}$ $where: (p > 0, q > 0)$ [G.B.T.U 2012, 15]

(44) Evaluate: (i) $\Gamma(3.5)$ (ii) $\beta\left(\frac{1}{2},\frac{1}{2}\right)$ (iii) $\beta(2,1) + \beta(1,2)$

(45) Evaluate: $\frac{\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{6}\right)}{\Gamma\left(\frac{2}{3}\right)} = (2)^{\frac{1}{3}}\sqrt{\pi}.$ [A.K.T.U. 2017]

[BEST OF LUCK]