Unit-I PDE

① Form the P.D.E. by eliminating the arbitrary functions from the following:

a) $z = f(x^2 - y^2)$

b) $z = \phi(x).\psi(y)$

c) $z = x + y + f(xy)$

② Solve the following P.D.E.

a) $y^2p - xyq = x(z - xy)$ (U.P.T.U – 2014)

b) $x^2p + y^2q = (x+y)z$ (U.P.T.U – 2015)

③ Solve the following P.D.E. $p + 3q = 5z + \tan(y - 3x)$ (A.K.T.U – 2017)

④ Solve $x^2p^2 + y^2q^2 = z^2$

⑤ Solve $p^2 - q^2 = x - y$

⑥ Solve: $(p^2 + q^2)y = qz$.

⑦ Use Cauchy's method of characteristics to solve:

$u_x - u_y = 0$; $u(x,0) = x$

⑧ Solve

a) $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 2\frac{\partial^3 z}{\partial x \partial y^2} = 0$ b) $R = a^2 t$

⑨ Solve the linear P.D.E. $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 0$

⑩ Solve the linear P.D.E.

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin(2x + 3y)$$

(11) $r + s - 2t = \sqrt{2x+y}$ (U.P.T.U 2015)

(12) Solve the linear P.D.E.

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos 4x \cos 3y + 3(2x+y)$$

(13) solve: $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x$.

(14) solve: $(D^2 + 5DD' + 6D'^2)z = \frac{1}{y-2x}$

(15) $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$ (A.K.TU – 2018)

(16) solve $\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x \partial y^2} - 6\frac{\partial^3 z}{\partial y^3} = \sin(x+y) + e^{3x+y}$

(U.P.T.U. – 2014)

(17) $(D^3 - 4D^2D' + 5DD'^2 - 2D'^3)z = e^{y+2x} + (y+x)^{1/2}$

(A.K.TU – 2017)

(18) Find a real function V of x and y, reducing to zero when $y=0$ and satisfying

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -4\pi(x^2+y^2)$$

(19) $(D^2 - 2DD' + 15D'^2)z = 12xy$

Unit-II Application of PDE

(1) Classify the following operators:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 u}{\partial t^2}$$

(2) Show that the equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ is hyperbolic

(3) Classify the P.D.E.

$$\frac{\partial^2 u}{\partial t^2} + t\frac{\partial^2 u}{\partial x \partial t} + x\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + 6u = 0$$

(4) Classify the following equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}$$

(5) Solve by the method of separation of variables:

$$4\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 3u, \quad u = 3e^{-x} - e^{-5x}, \text{ when } t = 0$$

(6) Solve the following equation by the method of seperation of variables.

$$\frac{\partial^2 u}{\partial x \partial t} = e^{-t}\cos x$$

(7) A string is stretched and fastened to two points l apart. Motion is started by the displacement of any point at a distance x from one end at a time t is given by

$$y(x,t) = A \sin\frac{\pi x}{l} \cos\frac{\pi ct}{l}.$$ (A.K.T.U.) (2018, 2017, 2013)

(8) A string is stretched between two fixed points $(0,0)$ and $(l,0)$ and released rest from the initial deflection given by

$$f(x) = \begin{cases} \left(\frac{2k}{l}\right)x, & 0 < x < l/2 \\ \left(\frac{2k}{l}\right)(l-x), & \frac{l}{2} < x < l \end{cases}$$

Find the deflection of the string at any time.

(9) A tightly stretched string with fixed end points $x=0$ and $x=\pi$ is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points an initial velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0.03\sin x - 0.04\sin 3x$

then find the displacement $y(x,t)$ at any point of string at any point t.

UNIT-III

① The mean of 200 items was 50. Later on it was discovered that two items were misread as 92 and 8 instead of 192 and 88. Find out the correct mean.

② Calculate the mode from the following frequency distribution:

Size(x):	4	5	6	7	8	9	10	11	12	13
Frequency(f):	2	5	8	9	12	14	14	15	11	13

③ An aeroplane flies along the four sides of a square at speeds of 100, 200, 300 and 400 km/hr respectively. What is the average speed of the aeroplane in its flight around the square?

④ Find the first four moments for the following individual series:

x	3	6	8	10	18

⑤ Calculate the $\mu_1, \mu_2, \mu_3, \mu_4$ for the following frequency distribution:

Marks	5-15	15-25	25-35	35-45	45-55	55-65
No. of students	10	20	25	20	15	10

⑥ In a certain distribution, the first four moments about the point $x=4$ are $-1.5, 17, -30$ and 308. Find the moments about mean & about origin. Also calculate β_1 and β_2. (UPTU-2014)

⑦ The first four moments of a distribution about the value '4' of the variable are $-1.5, 17, -30$ and 108. Find the moment about mean, about origin, β_1 and β_2. Also, find the moment about the point $x=2$. (AKTU-2017)

(8) What is moment generating function?

(9) Define skewness & Also the meaning of skewness.

(10) The sum of 20 observations is 300 and sum of their squares is 5000. The median is 15. Find the Karl Pearson's coefficient of skewness.

(11) The first four moments of a distribution about $x=4$ are 1, 4, 10 and 45 obtain the various chara. of the distribution on the basis of the given information. Comment on the type of nature of distribution. (AKTU – 2016, 2018)

(12) The following Table represents the height of a batch of 100 students. Calculate Kurtosis.

Height	59	61	63	65	67	69	71	73	75
No. of students	0	2	6	20	40	20	8	2	2

(AKTU – 2018)

(13) Fitting of the curve $y = ax^2 + \frac{b}{x}$. (AKTU – 2017)

(14) Fitting of the curve $y = ax + bx^2$ (AKTU – 2019)

(15) Define lines of regression. (AKTU – 2016)

(16) Angle between two lines of regression.

$$\tan\theta = \frac{1-r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$$ (AKTU – 2015, 17)

(17) For two random variables, x and y with the same mean the two regression equations are $y = ax + b$ & $x = \alpha y + \beta$ Show that $\frac{b}{\beta} = \frac{1-a}{1-\alpha}$. Find also the common mean

(18) The equations of two regression lines, obtained in a correlation analysis of 60 observations are:
$5x = 6y + 24$ and $1000y = 768x - 3608$.

UNIT- IV

(1) A and B throw alternately with a single die, A having the first throw. The person who first throw ace is to win. What are their respective chances of winning?

(2) A bag X contains 2 white & 3 Red balls and a bag Y contains 4 white and 5 Red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag Y.

(3) If the function $f(x)$ is defined by $f(x) = ce^{-x}, 0 \le x \le \infty$, find the value of c which changes $f(x)$ to a Probability density function.

(4) What is the expected value of the number of points that will be obtained in a single throw with an ordinary die? Find variance also.

(5) Mean and variance of the binomial distribution.

(6) If the probability of hitting a target is 10% and 10 shots are fired independently. What is the Probability that the target will be hit at least once? (AKTU-2018) (AKTU-2019)

(7) Six dice are thrown 729 times. How many times do you expect at least three dice to show a five or six?

(8) Mean and variance of poisson distribution. (AKTU-2018)

(9) Six coins are tossed 6400 times. Using the poisson distribution, determine the approximate probability of getting six heads x times.

(10) Mean and Variance of normal distribution.
(AKTU-2015, 18)

(11) The life of army shoes is normally distributed with Mean 8 months and standard deviation 2 months. If 1500 pairs are issued, how many pairs would be expected to need replacement after 12 months

[Given that $P(z \geq 2) = 0.0228$ & $z = \frac{x-\mu}{\sigma}$]

(12) Find mean, variance and third moment about mean for the binomial distribution.

Unit - V.

(1) A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased.

(2) A machine produced 16 defective articles in a batch of 500. After overloading it produced 3 defective in a batch of 100. Has the machine improved?

(3) A random sample of 900 members has a mean 3.4 cms. Can it be reasonably regarded as a sample from a large population of mean 3.2 cms and SD 2.3 cms?

(4) Samples of sizes 10 & 14 were taken from two normal populations with S.D. 3.5 and 5.2. The sample means were found to be 20.3 and 18.6. Test whether the means of two populations are same at 5% level.

(5) The following Table gives the no. of accidents that took place in an industry varies days of the week. Test of accidents are uniformly distributed over the week.

Day	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents	14	18	12	11	15	14

(6) A die is thrown 276 times and the results of these throws are given below:

No. appeared on the die	1	2	3	4	5	6
Frequency	40	32	29	59	57	59

(A.K-T.U-2019)

(7) Analysis of Variance (ANOVA).

(8) If the average fraction defective of a large sample of a product is 0.1537, Calculate the control limits given that sub-group size is 2000.

(9) Write a short note on 'ANOVA'.

(10) Write a short notes on:

I. Null hypothesis

II. Alternative hypothesis

III. Level of significance