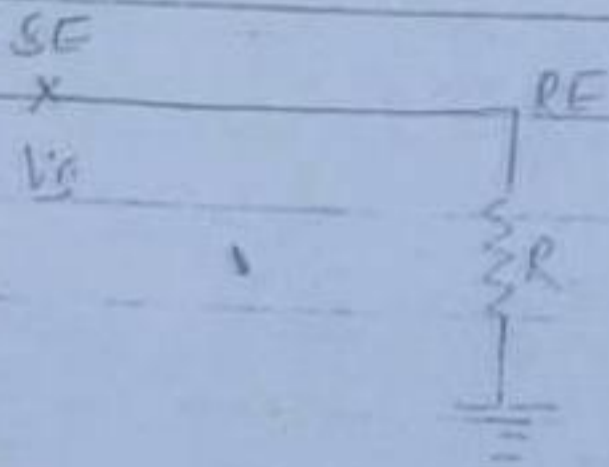


# Travelling Waves:

Condition 1: 'R'

receiving end of T.L terminated by resistance



$$V'' = V + V' \quad I'' = I + I'$$

$$I = V/Z_c \quad I' = -V'/Z_c \quad I'' = V''/R$$

$$V'' \rightarrow V, \quad V' \rightarrow V$$

Expressing transmitted voltage  $V''$  in terms of incident voltage  $V$  and transmitted current  $I''$  in terms of incident current  $I$ .

$$\Rightarrow V''/R = \frac{+V}{Z_c} + \left( \frac{-V'}{Z_c} \right)$$

replacing  $V'$  by  $(V'' - V)$

$$V''/R = \left( \frac{+V}{Z_c} \right) + \left( \frac{V'' - V}{Z_c} \right)$$

$$\frac{V''}{R} \left[ \frac{1}{R} + \frac{1}{Z_c} \right] = \frac{2V}{Z_c}$$

$$V'' = V \cdot \frac{2R}{R+Z_c} \quad \text{--- (1)}$$

• Transmitted voltage coefficient  $T_V = \frac{V''}{V} = \frac{2R}{R+Z_c}$

Similarly from eq<sup>n</sup> (2)

$$I'' R = I Z_c \left( \frac{2R}{R+Z_c} \right)$$

$$I'' = I \left( \frac{2Z_c}{R+Z_c} \right)$$

• Transmitted current coefficient

$$T_I = \frac{I''}{I} = \frac{2Z_c}{R+Z_c}$$

Expressing reflected voltage ( $V'$ ) in terms of incident voltage ( $V$ ) and reflected current ( $I'$ ) in terms of incident current ( $I$ ).

$$\frac{V''}{R} = \frac{+V}{Z_c} + \frac{(-V')}{Z_c}$$

$$\frac{V+V'}{R} = \frac{+V}{Z_c} + \frac{(-V')}{Z_c}$$

$$V \left[ \frac{1}{R} - \frac{1}{Z_c} \right] = -V' \left[ \frac{1}{R} + \frac{1}{Z_c} \right]$$



$$\frac{Z_c + R}{Z_c - R} V = -V' \frac{R + Z_c}{R - Z_c} \quad \text{--- (2)}$$

• reflection coefficient of voltage  $\left| \rho_v = \frac{V'}{V} = \frac{R - Z_c}{R + Z_c} \right|$

similarly using eqn (2)

$$V' = V \left( \frac{R - Z_c}{R + Z_c} \right)$$

$$-I' Z_c = I Z_c \left( \frac{R - Z_c}{R + Z_c} \right)$$

• reflection coefficient of current  $\left| \rho_i = \frac{I'}{I} = \frac{Z_c - R}{Z_c + R} \right|$

|            |                           |
|------------|---------------------------|
| $\tau_v =$ | $\frac{2R}{Z_c + R}$      |
| $\tau_i =$ | $\frac{2Z_c}{Z_c + R}$    |
| $\rho_v =$ | $\frac{R - Z_c}{Z_c + R}$ |
| $\rho_i =$ | $\frac{Z_c - R}{Z_c + R}$ |
| (c)        |                           |

condition 2: Receiving end of T.L terminated by underground cable with impedance ( $Z_1$ ).

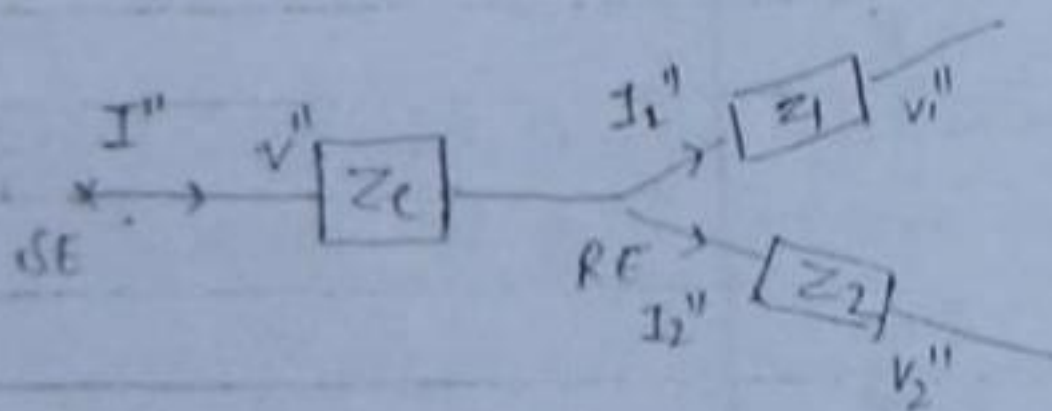
$$\bullet \tau_v = \frac{2Z_1}{Z_c + R + Z_1} \rightarrow \text{replaces } R \text{ by } Z_1$$

$$\bullet \tau_r = \frac{2Z_c}{Z_c + Z_1}$$

$$\bullet \rho_v = \frac{Z_1 - Z_c}{Z_c + Z_1}$$

$$\bullet \rho_r = \frac{Z_c - Z_1}{Z_c + Z_1}$$

condition 3: Receiving end of T.L line forming a T junction.



+  $Z_1, Z_2$  connected in parallel.

$$V_1'' = V_2'' = V''$$

$$I'' = I_1'' + I_2''$$

$$\Rightarrow I'' = I + I'$$

$$\Rightarrow I_1'' + I_2'' = I + I'$$



Expressing transmitted voltage ( $V''$ ) in terms of incident voltage ( $V$ ), and transmitted current ( $I''$ ) in terms of incident current ( $I$ ):-

$$I_1'' + I_2'' = I + I'$$

$$\Rightarrow \frac{V_1''}{Z_1} + \frac{V_2''}{Z_2} = \frac{V}{Z_c} + \left( \frac{-V'}{Z_c} \right)$$

$$\Rightarrow \frac{V''}{Z_1} + \frac{V''}{Z_2} = \frac{V}{Z_c} - \frac{V'}{Z_c}$$

$$\Rightarrow V'' \left( \frac{1}{Z_1} + \frac{1}{Z_2} \right) = \frac{V}{Z_c} - \frac{V'}{Z_c}$$

$$\Rightarrow V'' \left[ \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_c} \right] = \frac{2V}{Z_c}$$

$$V'' = V \frac{2/Z_c}{\left[ \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_c} \right]} \quad \text{--- (1)}$$

Transmitted  
• reflection coefficient of voltage

$$T_V = \frac{V''}{V} = \frac{2/Z_c}{\left[ \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_c} \right]}$$

now using equat<sup>n</sup> (1)

$$I'' Z_1 = I Z_c \frac{2/Z_c}{\left[ \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_c} \right]}$$

• Transmitted coefficient of current  $\tau_I = \frac{I''}{I} = \frac{2/Z_1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_c}}$

$\therefore \tau_{I_2} = \frac{I_2''}{I} = \frac{2/Z_2}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_c}}$

• Expressing reflected voltage in terms of incident voltage and reflected current in terms of incident current

$$I_1'' + I_2'' = I_0 + I'$$

$$\Rightarrow \frac{V_1''}{Z_1} + \frac{V_2''}{Z_2} = \frac{V}{Z_c} + \left( \frac{-V'}{Z_c} \right)$$

$$\frac{V''}{Z_1} + \frac{V''}{Z_2} = \frac{V}{Z_c} - \frac{V'}{Z_c}$$

$$\Rightarrow V'' \left( \frac{1}{Z_1} + \frac{1}{Z_2} \right) = \frac{V}{Z_c} - \frac{V'}{Z_c}$$

$$\Rightarrow (V + V') \left( \frac{1}{Z_1} + \frac{1}{Z_2} \right) = \frac{V}{Z_c} - \frac{V'}{Z_c}$$

$$\Rightarrow V \left( \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_c} \right) = \frac{V}{Z_c} - \frac{V'}{Z_c} = \frac{V}{Z_2}$$



$$\Rightarrow V' = V \left( \frac{\frac{1}{Z_c} - \frac{1}{Z_1} - \frac{1}{Z_2}}{\frac{1}{Z_c} + \frac{1}{Z_1} + \frac{1}{Z_2}} \right) \quad \text{--- (a)}$$

|                                  |   |
|----------------------------------|---|
| reflected coefficient of voltage | $S_v = \frac{V'}{V} = \left[ \frac{\frac{1}{Z_c} - \frac{1}{Z_1} - \frac{1}{Z_2}}{\frac{1}{Z_c} + \frac{1}{Z_1} + \frac{1}{Z_2}} \right]$ |
|----------------------------------|---|

Similarly, from eqn (a)

$$-I' \cdot Z_c = I \cdot Z_c \left( \frac{\frac{1}{Z_c} - \frac{1}{Z_1} - \frac{1}{Z_2}}{\frac{1}{Z_c} + \frac{1}{Z_1} + \frac{1}{Z_2}} \right)$$

$$\Rightarrow I' = I \left( \frac{\frac{1}{Z_1} + \frac{1}{Z_2} - \frac{1}{Z_c}}{\frac{1}{Z_c} + \frac{1}{Z_1} + \frac{1}{Z_2}} \right)$$

|                                  |   |
|----------------------------------|---|
| reflected coefficient of current | $S_i = \frac{I'}{I} = \left[ \frac{\frac{1}{Z_1} + \frac{1}{Z_2} - \frac{1}{Z_c}}{\frac{1}{Z_c} + \frac{1}{Z_1} + \frac{1}{Z_2}} \right]$ |
|----------------------------------|---|

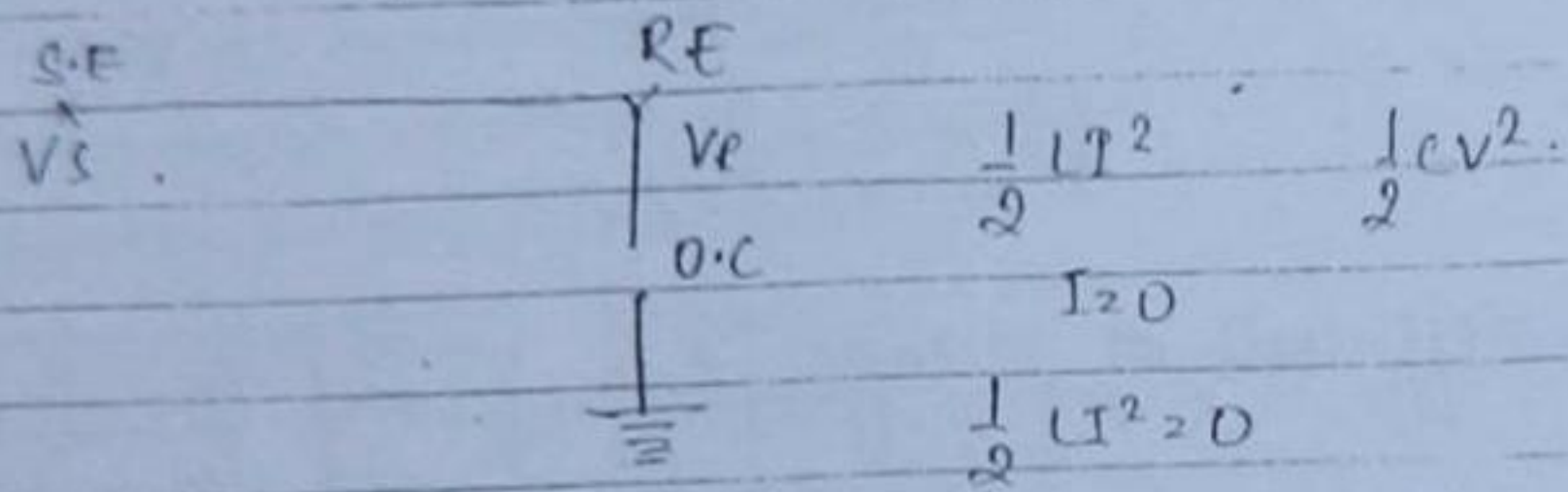
$$C_v = \left( \frac{2/Z_c}{\frac{1}{Z_c} + \frac{1}{Z_1} + \frac{1}{Z_2}} \right)$$

$$C_i = \left( \frac{2/Z_1 \text{ or } 2/Z_2}{\frac{1}{Z_c} + \frac{1}{Z_1} + \frac{1}{Z_2}} \right)$$

$$S_v = \left( \frac{\frac{1}{Z_c} - \frac{1}{Z_1} - \frac{1}{Z_2}}{\frac{1}{Z_c} + \frac{1}{Z_1} + \frac{1}{Z_2}} \right)$$

$$S_i = \left( \frac{\frac{1}{Z_1} + \frac{1}{Z_2} - \frac{1}{Z_c}}{\frac{1}{Z_c} + \frac{1}{Z_1} + \frac{1}{Z_2}} \right)$$

Condition 4: Receiving end of T.L is open circuited.



When the receiving end of T.L is open circuited ( $I=0$ ), electromagnetic energy stored by inductor in magnetic field is equal.  $\left[ \frac{1}{2} LI^2 = 0 \right]$ .

According to law of conservation of energy, energy cannot be destroyed but only can be converted from one form to another form. i.e. the entire electromagnetic energy is converted into electrostatic energy stored by capacitor in the electric field.

The increase in electrostatic energy increases the voltage. Let the voltage be increased by 'e' volts.

$$V \uparrow \rightarrow 'e' \text{ volts.}$$

$$\text{so } \frac{1}{2} LI^2 = \frac{1}{2} Ce^2$$

$$I^2 = e^2 / LC$$

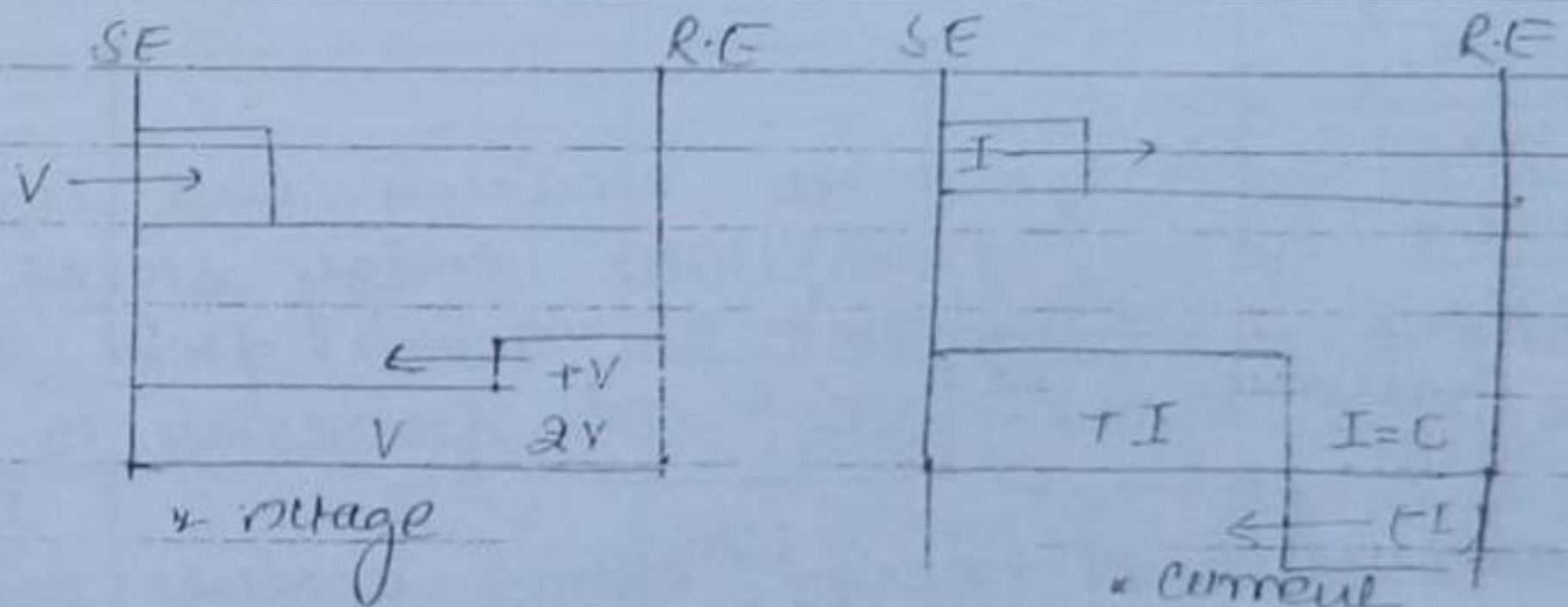


incident current  
↓  
1

$$P = I^2 Z_c^2$$

$$P = I \cdot Z_c = V$$

When the receiving end is open circuit the voltage is increased by 'V' where V is incident or voltage at sending end.



$$\text{reflection coefficient } R_V = +V/V = 1$$

when receiving end is O.C

$$\text{reflection coefficient } R_I = \frac{-I}{I} = -1$$

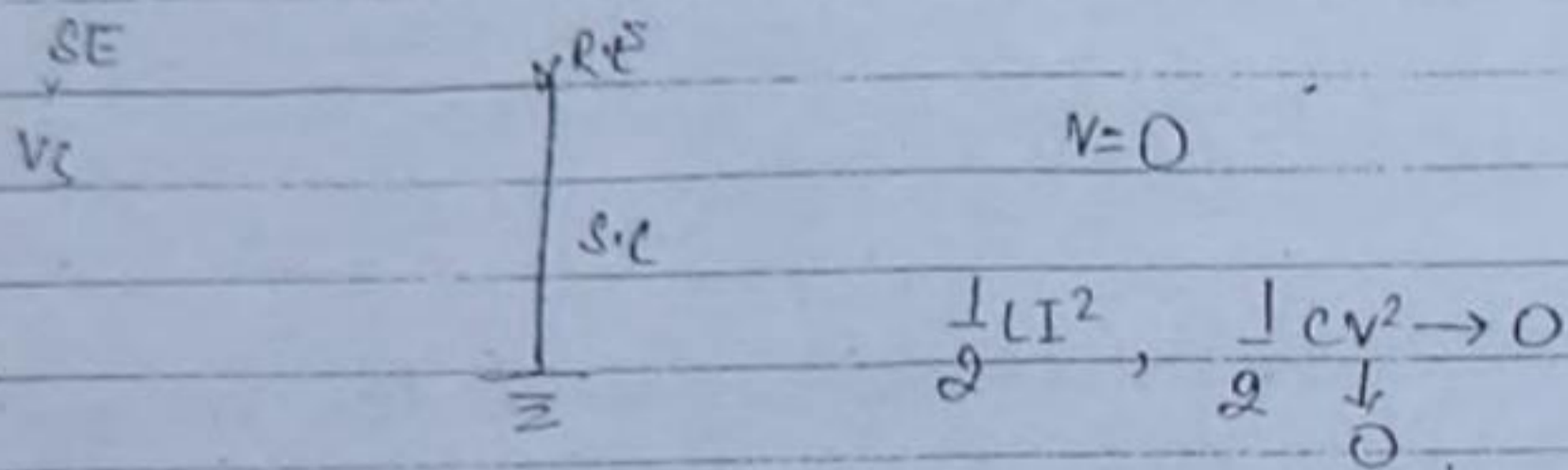
$$T_V = \frac{V'}{V} = \frac{V+V}{V} = \frac{V+V}{V} = 2$$

$$T_V = 2$$

$$T_I = 0/I = 0$$

$$T_I = 0$$

Conditions: Receiving end to T.L short-circuited.



$$\frac{1}{2} LI^2$$

EMF  $\uparrow$

$I \uparrow$

$I$  Amp

when receiving end is S.C.  $V=0$   
 electrostatic energy stored by capacitor in electric field is equal to  $\frac{1}{2} CV^2 = 0$ . According to law

of conservation of energy, energy is never destroy but only is converted from one form to another form. i.e. entire electrostatic energy is converted into electromagnetic energy. As a result electromagnetic energy increases and current increase.

Let the current be increased by  $I$  Amp

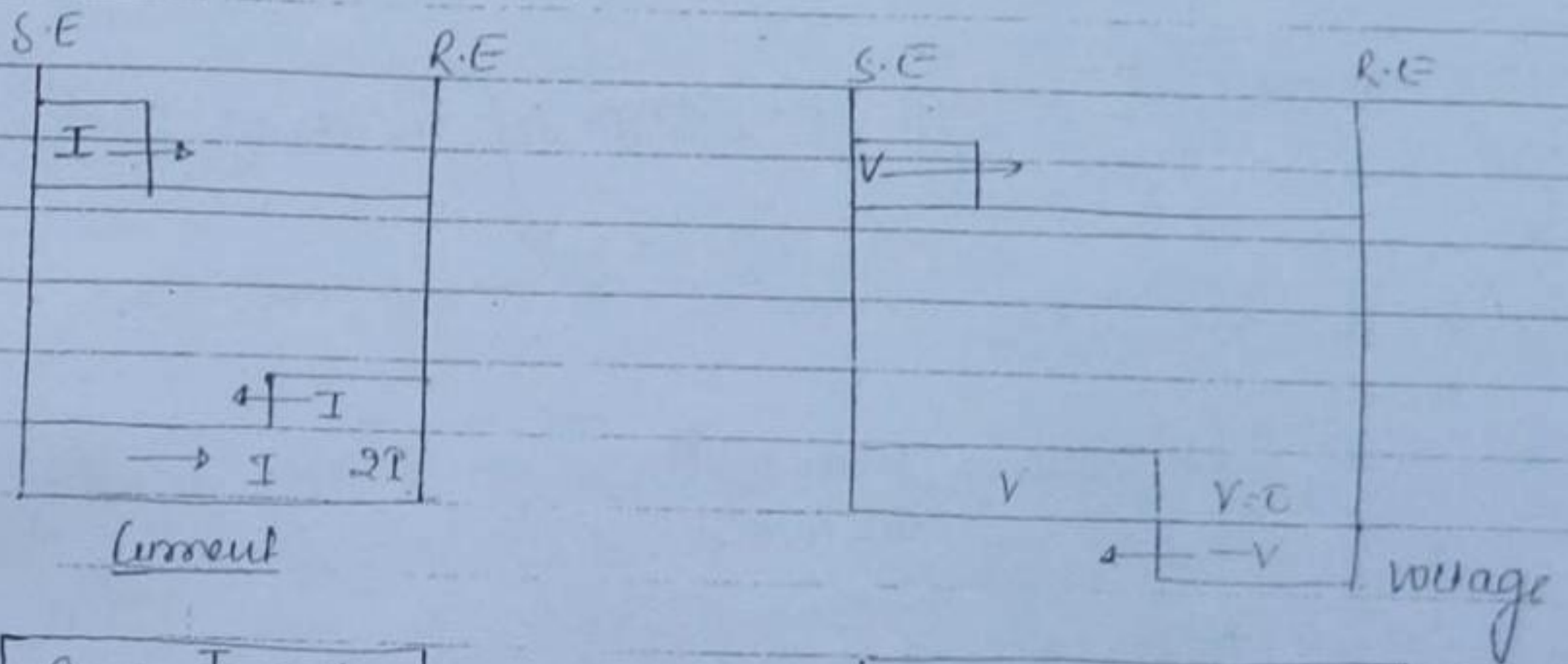
$$\frac{1}{2} LI^2 = \frac{1}{2} CV^2$$

$$\Rightarrow I^2 = \frac{V^2}{L/C} = \frac{V^2}{Z_c^2}$$

$$I = \frac{V}{Z_c} = I$$



The current  $i$  increases by  $I$  Amp, where  $I$  is incident current or current flowing from sending end to the receiving end of the T.L.



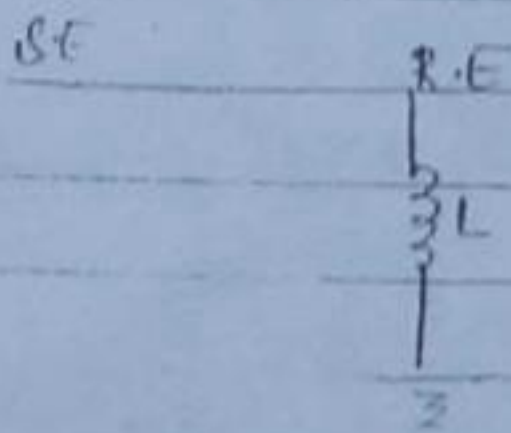
$$S_I = \frac{I}{I} = 1$$

$$Z_I = \frac{2\Omega}{I} = 2$$

$$S_V = \frac{-V}{V} = -1$$

$$Z_V = \frac{C}{V} = C$$

Condition 6<sup>o</sup>: Receiving end of T.L is terminated by inductor 'L'



$$V'' = V + V'$$

$$I'' = I + I'$$

$$I = V/Z_c \quad ; \quad I' = -V'/Z_c$$

$$I'' = \frac{1}{L} \int V''(t) dt$$

Expressing transmitted voltage in terms of incident voltage  $V$

$$I'' = I + I'$$

$$\Rightarrow \frac{1}{L} \int V''(t) dt = \frac{V}{Z_c} + \frac{(-V')}{Z_c}$$

$$\Rightarrow \frac{1}{L} \int V''(t) dt = \frac{V}{Z_c} - \left( \frac{V' - V}{Z_c} \right)$$

$$\Rightarrow \frac{1}{L} \int V''(t) dt = \frac{2V}{Z_c} - \frac{V'}{Z_c}$$

$$\Rightarrow \frac{1}{L} \int V''(t) dt + \frac{V'}{Z_c} = \frac{2V}{Z_c}$$

Applying Laplace transform.

$$\frac{1}{L} \left\{ \frac{V''(s)}{s} \right\} + \frac{V'(s)}{Z_c} = \frac{2V}{sZ_c}$$

$$V'(s) \left[ \frac{1}{sL} + \frac{1}{Z_c} \right] = \frac{2V}{sZ_c}$$



$$\Rightarrow V''(s) \cdot \left\{ \frac{sL + Z_c}{sZ_c} \right\} = \frac{2V}{sZ_c}$$

$$V''(s) = \frac{2VL}{sL + Z_c}$$

dividing by L.

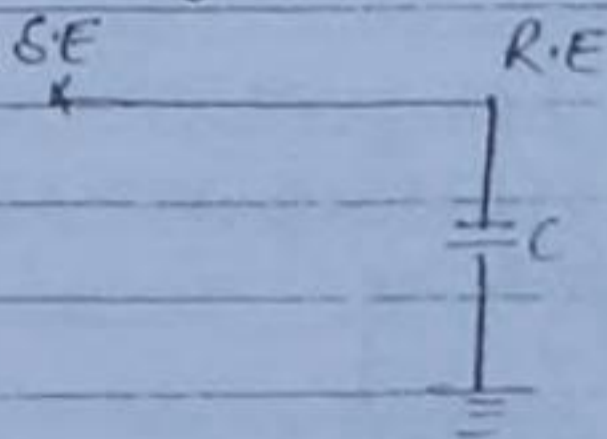
$$V''(s) = \frac{2V}{s + \frac{Z_c}{L}}$$

Applying inverse Laplace transform.

$$\Rightarrow V''(t) = 2V \cdot e^{-\frac{Z_c}{L} \cdot t}$$

When T.L is terminated by inductor the incident transmitted voltage decreases exponentially due to presence of inductor.

Condition 2: Receiving end of T.L terminated by capacitance



$$V'' = V + V'$$

$$I'' = I + I'$$

$$I = V/Z_c; I' = -V'/Z_c$$

$$I'' = C \frac{dV''(t)}{dt}$$

$$I + I' = C \frac{dV''(t)}{dt}$$

$$\frac{V}{Z_c} = \frac{V'}{Z_c} = C \frac{dV'(t)}{dt} \Rightarrow \frac{C dV''(t)}{dt} + \frac{V''(t)}{Z_c} = \frac{2V}{Z_c}$$

applying Laplace transform

$$\frac{V(s)}{Z_c} - \frac{V'(s)}{Z_c} = \frac{C}{s}$$

$$\Rightarrow C \left\{ sV''(s) \right\} + \frac{V''(s)}{Z_c} = \frac{2V}{sZ_c}$$

$$\Rightarrow V''(s) \left\{ \frac{Cs + 1}{Z_c} \right\} = \frac{2V}{sZ_c}$$

$$\Rightarrow V''(s) \left\{ \frac{sCZ_c + 1}{Z_c} \right\} = \frac{2V}{sZ_c}$$

$$\Rightarrow \underline{V''(s)} = \frac{2V}{s(sCZ_c + 1)}$$

dividing by  $CZ_c$

$$V''(s) = \frac{2V/CZ_c}{s(s + \frac{1}{CZ_c})}$$

using partial fraction

$$A \cdot \frac{1}{s} = \frac{2V}{CZ_c}$$

$$'s' : A + B = 0$$

$$B = -A = -2V$$



$$V''(s) = \frac{2V}{s} - \frac{2V}{s + \frac{1}{CZ}}$$

Applying inverse Laplace transform:-

$$V''(t) = 2V - 2V \cdot e^{-1/CZ \cdot t}$$

$$V''(t) = 2V \left\{ 1 - e^{-1/CZ \cdot t} \right\} \text{ volt}$$

due to discharging effect.

When T.L. terminated by capacitor is decrease exponentially.

Ques 7:

An inductance of 800 mH connect two sect<sup>n</sup> of T.L each having surge impedance of 350  $\Omega$ . a 500

A 500 kV, 2  $\mu$ sec rectangular surge travels along the line towards the inductance. The max<sup>m</sup> value of transmitted wave is.  $V'' = 2V e^{-Z_0/Lt}$

$$V'' = 2V \cdot 500 \times e^{-\left(\frac{350}{800 \times 10^{-6}}\right) (2 \times 10^{-6})}$$

$$V'' = 416.86 \text{ kV}$$

4) A 500kV, 2μsec rectangular wave surge on T.L having surge impedance of 350Ω approaches a generating station at which the concentrated earth capacitance is 3000pF. The max value of the transmitted wave is

$V$

$$V'' = 2V \left( 1 - e^{-t/\tau_c} \right)$$

$$V'' = 2 \times 500 \left[ 1 - e^{-\left( 2 \times 10^{-6} / 350 \times 3000 \times 10^{-12} \right)} \right]$$

$$V'' = 850 \text{ kV}$$

5) A voltage surge of 60kV travelling in a line of natural impedance 500Ω, arrives at a junction with two T.L of impedance, 650Ω, 250Ω. The surge voltage and current transmitted into each branch of line are?

$$Z_c = 500 \Omega$$

$$Z_1 = 650, \quad Z_2 = 250$$

$$V'' = V \left( \frac{2/Z_c}{\frac{1}{Z_1} + \frac{1}{Z_2} + 1} \right)$$



$$I_1'' = \frac{V_1''}{Z_1} = \frac{31.84 \times 10^3}{650} = 48.97 \text{ Amp.}$$

$$I_2'' = \frac{V_2''}{Z_2} = \frac{31.84 \times 10^3}{250} = 127.4 \text{ Amp}$$

4) A voltage surge of 15KV travels along a cable towards a junction with an overhead T.L. The inductance and capacitance of overhead T.L. cable are 0.3mH, 0.4uF. The inductance and capacitance of overhead T.L. are 1.5mH, and 0.12uF. The increase in voltage at the junction due to surge is?

$$Z_{c \text{ cable}} = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.3 \times 10^{-3}}{0.4 \times 10^{-6}}} = 27.38 \Omega$$

$$Z_{c \text{ T.L.}} = \sqrt{\frac{1.5 \times 10^{-3}}{0.12 \times 10^{-6}}} = 353 \Omega$$

$$V'' = \frac{2R}{Z_c + R} \Rightarrow \frac{2Z_{c \text{ cable}}}{Z_c + Z_{c \text{ cable}}} \text{ as surge travel through cable}$$

$$\frac{2 \times 27.38}{353 + 27.38} \cdot 15 \times 10^3 = \frac{2 \times 27.38}{380.38} \times 15 \times 10^3$$

$$= 21.84 \text{ kV}$$

A-  $\phi$  overhead conductors, in equilateral configuration the characteristic impedance is  $300\Omega$ . What should be load impedance show that reflections do not occur in T.L.

TELEGRAPHER | WAVE EQUATION !

$$\frac{\partial^2 e}{\partial x^2} = RGe + (RC + LG) \frac{\partial e}{\partial t} + LC \frac{\partial^2 e}{\partial t^2}$$

$$\frac{\partial^2 i}{\partial x^2} = RGi + (RC + LG) \frac{\partial i}{\partial t} + LC \frac{\partial^2 i}{\partial t^2}$$



# SURGE IMPEDENCE :-

Transmission line is known as surge impedance of lossless  
for a lossless T.L.  $Z_c = Z_s$

$$Z_c = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \rightarrow \text{characteristic impedance}$$

$$R=0; G=0$$

$$Z_s = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}$$

$$Z_s = \sqrt{\frac{L}{C}}$$

characteristic impedance of overhead line is 400  $\Omega$   
• underground cable is 40  $\Omega$

## Flat line OR INFINITE LINE

A lossless transmission line terminated with its characteristic impedance is known as flat line or infinite line. The phase angle of characteristic impedance of T.L is  $\phi = (-15^\circ)$

Surge impedance loading / characteristic impedance load

CIL and S.I.L refers to MW, MVA, ~~W~~MVAR of load connected at the receiving end of T.L, when the T.L do not have losses. S.I.L

$$SIL = \frac{V_s V_R}{Z_c}$$

If the T.L have losses

$$CIL = \frac{V_s V_R}{Z_c}$$

in terms of ABCD constant

$$SIL/CIL = \frac{V_s V_R}{B}$$

Relation b/w characteristic impedance, open-circuit impedance, short circuit impedance

$$V_s = AV_R + BI_R$$
$$I_s = CV_R + DI_R$$

Receiving end is O.C

$$I_R = 0$$



$$\Rightarrow V_C = A \cdot V_{R0} \Rightarrow A = V_C / V_{R0}$$

$$I_S = C \cdot V_{R0} \Rightarrow C = I_S / V_{R0}$$

$$\boxed{A/C = V_C / I_S} \quad \text{--- (1)}$$

Receiving end is S.C.  $V_R = 0$ .

$$V_C = B I_R \Rightarrow B = V_C / I_R$$

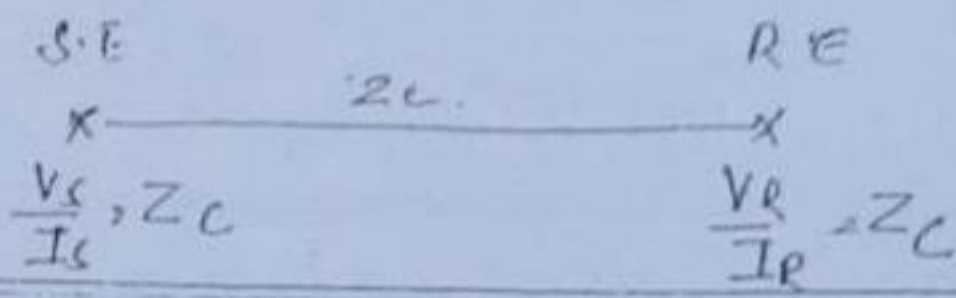
and  $D = I_S / I_R$ .

$$\boxed{B/D = \frac{V_C}{I_S}} \quad \text{--- (2)}$$

multiplying eq<sup>n</sup> (1) and (2)

$$\frac{V_C}{I_S} = \sqrt{\frac{A \cdot B}{C \cdot D}} \quad A = D$$

$$\boxed{\frac{V_C}{I_S} = \sqrt{\frac{B}{C}}}$$



$$Z_C = \sqrt{\frac{B}{C}} = \sqrt{\frac{Z_{sc}}{Y_{oc}}}$$

$$Z_C = \sqrt{Z_{sc} Z_{oc}}$$

### Questions:

1) The surge impedance loading of 400KV T.L is

$$\frac{V_s \times V_r}{B} = \frac{V_s \cdot V_r}{Z_C} = \frac{400 \times 400}{400}$$

$$Z_s = 400 \text{ MW}$$

2) Surge impedance loading of a 220KV of T.L is

$$\frac{220 \times 220}{400} = 121 \text{ MW}$$

# A T.L can be loaded more than surge impedance loading or less than surge impedance loading



# Condition 1:

$$\text{Loading} > S.I.L$$

1) Current increases.

2) load impedance ( $Z_L$ )  $< Z_c$ .

3) As  $\frac{I_L^2}{2} > \frac{I_C^2}{2}$ , so the power factor is  
lagging

4)  $V_R < V_S$

# Condition 2:

$$\text{Loading} < S.I.L$$

1) Current decrease.

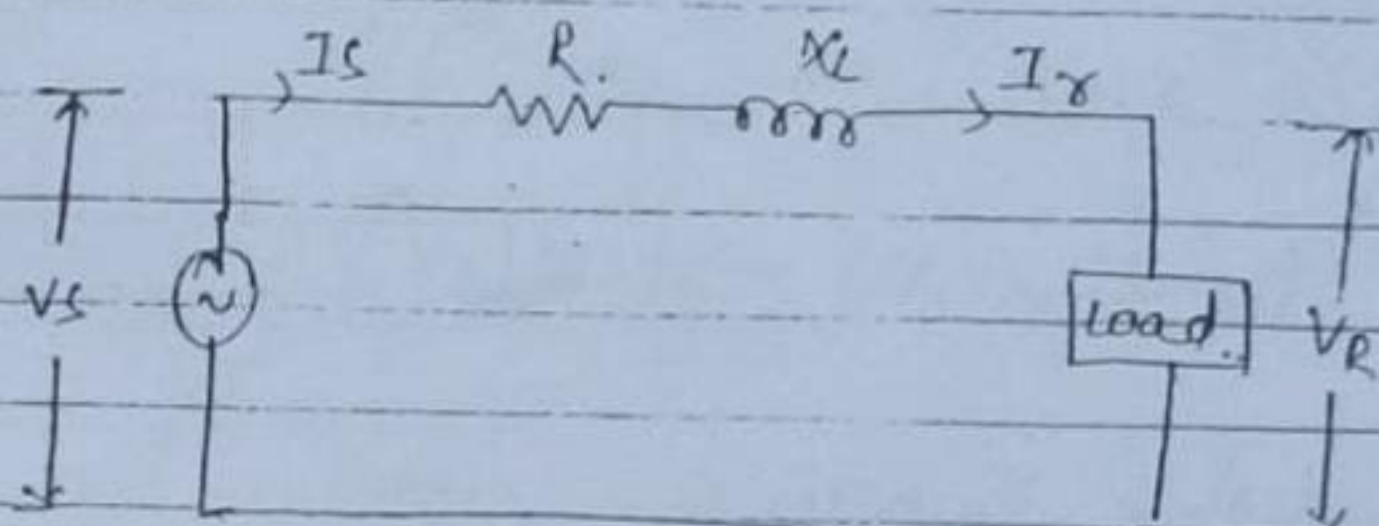
2) load impedance ( $Z_L$ )  $> Z_c$

3) As  $\frac{I_L^2}{2} < \frac{I_C^2}{2}$ , so the power factor is  
leading

4)  $V_R > V_S$ .

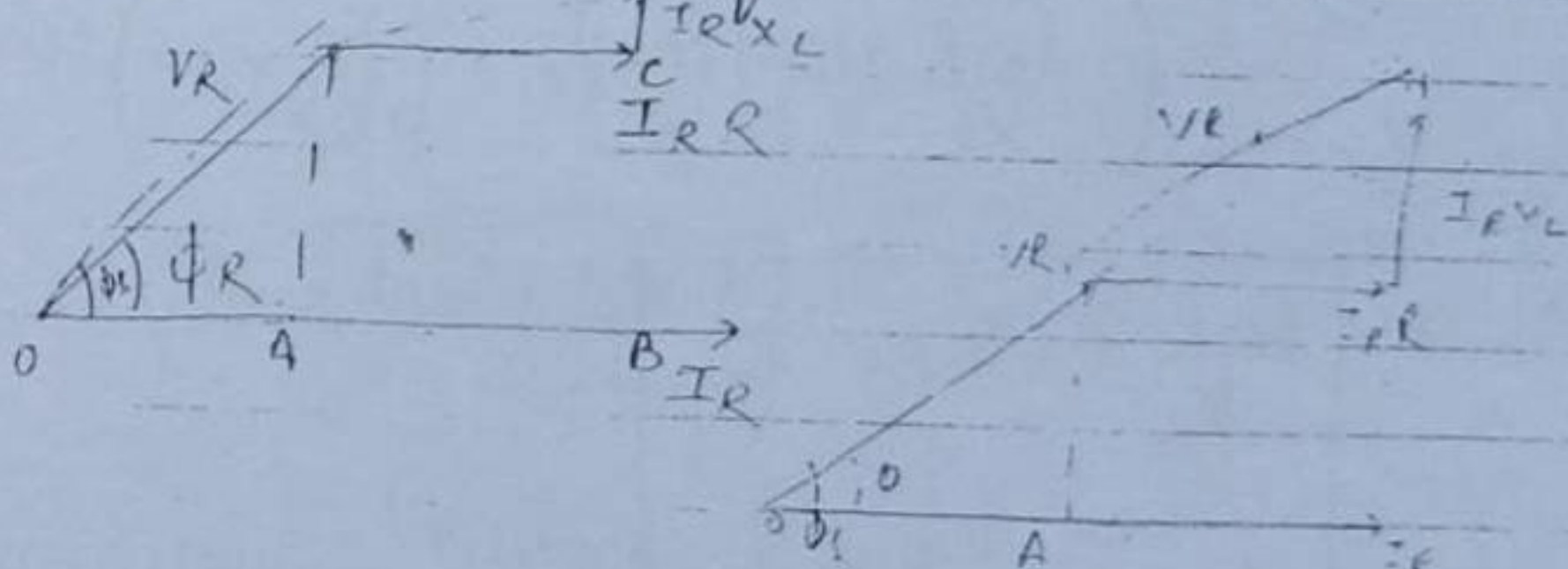
Sw general TL is loaded more than surge impedance loading. Surge impedance also called Natural impedance.

# CONDITION FOR ZERO REGULATION OF T.L.:-



$V_s \sim 0$

consider R.E current  $I_R$  as reference vector.



$$V_s^2 = OD^2 = OB^2 + BD^2$$

$$V_s^2 = (OA + AB)^2 + (BC + CD)^2$$

$$OB = V_s \cos \phi_s = OA + AB = V_R \cos \phi_R + I_R R \quad \text{--- (1)}$$

$$BD = V_s \sin \phi_s = BC + CD = V_R \sin \phi_R + I_R X_L \quad \text{--- (2)}$$

$$(1)^2 + (2)^2$$

$$V_s^2 = V_R^2 + I_R^2 (R^2 + X_L^2)$$