SVNIET BBK [2019]

(MATH-1I) UNIT-2 (KAS-203) (ASSIGNMENT)

(1) Define Bounded function with examples.

(2) Examine the convergence of the improper integrales:

(a) $\int_1^\infty \frac{1}{x}\,dx$ (b) $\int_1^\infty \frac{dx}{\sqrt{x}}$ (c) $\int_1^\infty \frac{dx}{x^{\frac{3}{2}}}$ (d) $\int_0^\infty \frac{1}{1+x^2}\,dx$. (e) $\int_a^\infty \frac{x}{1+x^2}\,dx$

(f) $\int_0^\infty \frac{1}{(1+x)^3}\,dx$ (g) $\int_0^\infty \frac{1}{x^2+4a^2}\,dx$ (h) (e) $\int_3^\infty \frac{1}{(x-2)^2}\,dx$ (f) (e) $\int_{\sqrt{2}}^\infty \frac{1}{x\sqrt{x^2-1}}\,dx$

(3) Examine the convergence the integrales: (a) $\int_1^\infty xe^{-x}dx$ (b) $\int_0^\infty x^2e^{-x}dx$

(c) $\int_0^\infty xe^{-x^2}dx$ (d) $\int_0^\infty x^3e^{-x^2}dx$ (e) $\int_0^\infty x\,sinx\,dx$

(4) Examine the convergence the integrales:

(a) $\int_1^\infty \frac{dx}{(1+x)\sqrt{x}}$ (b) $\int_2^\infty \frac{dx}{x\,logx}$ (c) $\int_0^\infty e^{-x}\,sinx\,dx$ (d) $\int_0^\infty e^{-ax}\,cosbx\,dx$

(5) Examine the convergence the integrales:

(a) $\int_1^\infty \frac{dx}{x(1+x)}$ (b) $\int_1^\infty \frac{dx}{x^2(x+1)}$ (c) $\int_1^\infty \frac{tan^{-1}x}{x^2}\,dx$ (d) $\int_0^\infty e^{-\sqrt{x}}\,dx$

(6) Examine the convergence the integrales:

(a) $\int_{-\infty}^0 e^{2x}\,dx$ (b) $\int_{-\infty}^0 \frac{dx}{p^2+q^2x^2}$ (c) $\int_{-\infty}^0 e^{-x}\,dx$ (d) $\int_{-\infty}^0 sinhx\,dx$

(7) Examine the convergence the integrales:

(a) $\int_{-\infty}^\infty e^{-x}\,dx$ (b) $\int_{-\infty}^\infty \frac{1}{1+x^2}\,dx$ (c) $\int_{-\infty}^\infty \frac{1}{e^x+e^{-x}}\,dx$

(8) Test the convergence the integrales: (a) $\int_0^1 \frac{dx}{\sqrt{x}}$ (b) $\int_0^1 \frac{dx}{x^2}$ (c) $\int_1^2 \frac{x}{\sqrt{x-1}}\,dx$

(9) Examine the convergence the integrales:

(a) $\int_0^1 logx\,dx$ (b) $\int_0^{\frac{1}{e}} \frac{1}{x(logx)^2}\,dx$ (c) $\int_1^2 \frac{1}{x\,logx}\,dx$

(10) Examine the convergence the integrales:

(a) $\int_0^a \frac{1}{\sqrt{a-x}}\,dx$ (b) $\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx$ (c) $\int_0^{\frac{\pi}{2}} \frac{cosx}{\sqrt{1-sinx}}\,dx$

(11) Examine the convergence the integrales:

(a) $\int_{-1}^1 \frac{1}{x^2}\,dx$ (b) $\int_a^{3a} \frac{1}{(x-2a)^2}\,dx$

(12) Examine the convergence the integrales:

(a) $\int_0^4 \frac{1}{x(4-x)}\,dx$ (b) $\int_{-a}^a \frac{x}{\sqrt{a^2-x^2}}\,dx$ (c) $\int_0^\pi \frac{1}{1+cosx}\,dx$ (d) c) $\int_0^\pi \frac{1}{sinx}\,dx$

(13) (i) The improper integral $\int_a^b \frac{dx}{(x-a)^n}$ is convergent if and only if $n < 1$.

(ii) The improper integral $\int_a^b \frac{dx}{(b-x)^n}$ is convergent if and only if $n < 1$.

(14) The improper integral $\int_a^\infty \frac{1}{x^n}\,dx\,,(a > 0)$ is convergent if and only if $n > 1$.

(15) Define Beta and Gamma function. [G.B.T.U. 06, 08, (SUM) 08 , 2018]

(16) Prove that $\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$ [G.B.T.U. 10, G.B.T.U(SUM) 09 , AKTU2018]

(17) Prove that: $\Gamma(m)\Gamma\left(m+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{(2)^{2m-1}}\Gamma(2m)$, where m is positive integer. [G.B.T.U. 2013]

(OR) State and prove Duplication formula.

(18) Using Beta and Gamma functions, evaluate: (i) $\int_0^\infty x^{\frac{1}{4}}\,e^{-\sqrt{x}}dx$. [G.B.T.U. (SUM) 2008]

(ii) $\int_0^1 \left(\frac{x^3}{1-x^3}\right)^{\frac{1}{2}} dx$. [G.B.T.U. 07,14,18] (iii) $\int_0^\infty \frac{dx}{1+x^4}$ [G.B.T.U. 2012]

(19) Prove that :- $\beta(l,m).\beta(l+m,n).\beta(l+m+n,p) = \frac{\Gamma(l)\Gamma(m)\,\Gamma(n)\Gamma(p)}{\Gamma(l+m+n+p)}$ [G.B.T.U. 2008]

(20) Evaluate : $\int_0^1 \frac{1}{\sqrt{1+x^4}}\, dx$ [G.B.T.U. 2011]

(21) Prove that :− $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$ [G.B.T.U. 2009]

(22) Evaluate the integral $\int\int\int x^{l-1}y^{m-1}z^{n-1}\ dx\, dy\, dz$ where x,y,z are all positive but limit By the condition $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \le 1$. [G.B.T.U. (SUM) 2009, G.B.T.U. 2006, 2011]

(23) Apply Dirichlet's integral to find the mass of an octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, The density at any point being $\rho = kxyz$. [G.B.T.U. 2006, 2007, (C.O.)2011, 2015]

(24) Prove that : $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{sin\theta}} = \int_0^{\frac{\pi}{2}} \sqrt{sin\theta}\, d\theta = \pi$. [G.B.T.U. 2014]

(25) Show that the area bounded by the curve $x^n + y^n = a^n$ and the co-ordinate axis in the First quadrant $\frac{a^2\,\Gamma\left(\frac{1}{n}\right)^2}{2n\,\Gamma\left(\frac{2}{n}\right)}$. [G.B.T.U. (C.O) 2011]

(26)Find Area and mass contained in the first quadrant enclosed by the curve $\left(\frac{x}{a}\right)^\alpha + \left(\frac{y}{b}\right)^\beta = 1$ Where $\alpha > 0, \beta > 0$ given that density at any point$\rho(x,y)\ is\ k\sqrt{xy}$. [G.B.T.U. 2009]

(27) Prove that: $\int_0^\infty \sqrt{y}\ e^{-y^2}\, dy \times \int_0^\infty \frac{e^{-y^2}}{\sqrt{y}}\, dy = \frac{\pi}{2\sqrt{2}}$. [G.B.T.U. (C.O) 2011]

(28) Prove that: $\int_0^1 \frac{x^2\, dx}{\sqrt{1-x^4}} \times \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}$. [G.B.T.U. (SUM) 2008]

(29) Prove that: $\int_0^{\frac{\pi}{2}} \sqrt{tan\theta}\ d\theta = \int_0^{\frac{\pi}{2}} \sqrt{cot\theta}\ d\theta = \frac{\pi}{\sqrt{2}}$. [G.B.T.U. 2013]

(30) Find the volume contained in the solid region in the first octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. [G.B.T.U. 2014]

(31) Evaluate $\int\int\int_V\ e^{-(x+y+z)}\ dx\, dy\, dz$, where the region of integration is bounded by Planes $x = 0, y = 0, z = 0$. and $x + y + z = a.\ a > 0$. [G.B.T.U. (SUM) 2008]

(32) Evaluate $\Gamma\left(\frac{1}{4}\right)\,\Gamma\left(\frac{3}{4}\right)$. [G.B.T.U. (CO) 2013]

(33) Prove that $\iiint \frac{dx\, dy\, dz}{\sqrt{1-x^2-y^2-z^2}} = \frac{\pi^2}{8}$, the integral being extended to all positive values of the vairiables for which the expression is real. [G.B.T.U.,2014 , 2016]

(34) Evaluate $\int\int\int_R (x^2 + y^2 + z^2)\, dx\, dy\, dz$ where $R\ denotes\ the\ region\ bounded\ by$ $x = 0, y = 0,\ \ z = 0\ and\ x + y + z = a,\ (a > 0.)$ [G.B.T.U. (CO) 2013]

(35) Find the mass of solid $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r = 1$, the density at any point being $\rho = kx^{l-1}y^{m-1}z^{n-1}$ where x, y, z are all positive. [G.B.T.U. 2016]

(36) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the axis in $A, B\ and\ C$. Apply Dirichlet's integral to find the volume of the tetrahedron $OABC$. Also find its mass if the density at any point is $kxyz$. [12,18]

(37) Evaluate. $\Gamma\left(-\frac{3}{2}\right) = \frac{4}{3}\sqrt{\pi}$ [G.B.T.U. 2013]

(38) $\Gamma\left(-\frac{1}{2}\right) = -\,2\sqrt{\pi}$ [G.B.T.U. 2010]

(39) Evaluate. $\iiint\ (ax^2 + by^2 + cz^2)\, dx\, dy\, dz\ , Where\ V\ is\ the\ Region\ bounded\ by$ $x^2 + y^2 + z^2 \le 1$. [G.B.T.U. 2013]

(40) Evaluate:- (i) $\Gamma\left(-\frac{5}{2}\right)$ (ii) $\Gamma\left(-\frac{7}{2}\right)$.

(41) Compute $\iiint_V x^2\,dx\,dy\,dz$, over volume of tetrahedron bounded by $x = 0, y = 0, z = 0$ And $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. [G.B.T.U 2017]

(42) Evalute $\iiint_V x^2yz\,dx\,dy\,dz$, throughout the volume bounded by planes $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. [G.B.T.U 2017] .

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(43) Evaluate: (i) $\frac{\beta(m+1,n)}{\beta(m,n)}$ [G.B.T.U 2011] (ii) Evaluate: $\int_0^\infty e^{-x^2}\,dx$ [G.B.T.U 2011]

(82) Prove that: $\frac{B(p,\ q+1)}{q} = \frac{B(p+1,\ q)}{p} = \frac{B(p,\ q)}{p+q}$ $where: (p > 0, q > 0)$ [G.B.T.U 2012, 15]

(44) Evaluate: (i) $\Gamma(3.5)$ (ii) $\beta\left(\frac{1}{2},\frac{1}{2}\right)$ (iii) $\beta(2,1) + \beta(1,2)$

(45) Evaluate: $\frac{\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{6}\right)}{\Gamma\left(\frac{2}{3}\right)} = (2)^{\frac{1}{3}}\sqrt{\pi}.$ [A.K.T.U. 2017]

[BEST OF LUCK]